This exam was prepared by Stephanie Hays and Timothy Goldberg of LRU.
1. Margie’s car can go 32 miles on a gallon of gas, and gas currently costs $4 per gallon. How many miles can Margie drive on $20 worth of gas?

   (A) 64   (B) 128   (C) 160   (D) 320   (E) 640

2. A quadratic equation \( y = ax^2 + bx + c \) is known to pass through the points \((1, 6), (2, 17), \) and \((-4, 11)\). Find the sum of the coefficients.

   (A) 6   (B) 8   (C) 10   (D) 12   (E) none of the answers (A)–(D) is correct

3. In \( \triangle ABC \), \( D \) is a point on side \( \overline{AC} \) such that \( BD = DC \) and \( \angle BCD \) measures 70°. What is the measure of \( \angle ADB \)?

   (A) 100°   (B) 120°   (C) 135°   (D) 140°   (E) 150°

4. A popular musician opens sales for her upcoming concert at 10:00 am this morning. The number of tickets sold doubles every five minutes. This means at 10:00 am she has sold 1 ticket, at 10:05 am she has sold 2 tickets, at 10:10 am she has sold 4 tickets, at 10:15 am she has sold 8 tickets, and so on. If her concert is completely sold out by 11:00 am, approximately when were 10% of her tickets sold?

   (A) 10:06 am   (B) 10:15 am   (C) 10:30 am   (D) 10:45 am   (E) 10:55 am

5. The area of the largest triangle that can be inscribed in a semi-circle whose radius is \( r \) is:

   (A) \( r^2 \)   (B) \( r^3 \)   (C) \( 2r^2 \)   (D) \( 2r^3 \)   (E) none of the answers (A)–(D) is correct

6. A square of perimeter 20 units is inscribed in a square of perimeter 28 units, as shown below. What is the greatest distance between a vertex of the inner square and a vertex of the outer square?

   (A) \( \sqrt{58} \) units   (B) \( \frac{7\sqrt{5}}{2} \) units   (C) 8 units   (D) \( \sqrt{65} \) units   (E) 5\( \sqrt{3} \) units
7. It takes a boy riding his bicycle 9 minutes longer to deliver the newspapers to the homes on his route than it does his father driving in his car. Working together, they can deliver the papers in 20 minutes. How long would it take the boy working alone to deliver the papers?

(A) 29 minutes   (B) 36 minutes   (C) 45 minutes   (D) 54 minutes

(E) none of the answers (A)–(D) is correct

8. The sum of the first twenty-two odd numbers is \(x^2\). What is the value of \(x\)?

(A) 19   (B) 20   (C) 21   (D) 22   (E) none of the answers (A)–(D) is correct

9. With a single roll of a pair of dice, what is the probability that the sum of the two dice is 5?

(A) \(\frac{1}{6}\)   (B) \(\frac{1}{9}\)   (C) \(\frac{1}{18}\)   (D) \(\frac{5}{36}\)   (E) none of the answers (A)–(D) is correct

10. Andy’s lawn has twice as much area as Beth’s lawn and three times as much area as Carlos’ lawn. Carlos’ lawn mower cuts half as fast as Beth’s mower and one third as fast as Andy’s mower. If they all start to mow their lawns at the same time, who will finish first?

(A) Andy   (B) Beth   (C) Carlos   (D) Andy and Carlos tie for first.

(E) It’s a three-way tie.

11. Suppose \(y = x + 3\) and \((x + 2)^2 + (y - 4)^2 = 9\). Which of the following could be a value of \(3x + 5y\)?

(A) \(-1\)   (B) \(-11\)   (C) \(-17\)   (D) \(-23\)   (E) none of the answers (A)–(D) is correct

12. Find the equation of the line that is perpendicular to the line \(3x + 4y = -9\) and passes through the vertex of the parabola \(y = x^2 + 6x + 9\).

(A) \(y = -\frac{4}{3}x - 4\)   (B) \(y = -\frac{4}{3}x + 40\)   (C) \(y = \frac{4}{3}x + 4\)   (D) \(y = \frac{4}{3}x + 32\)

(E) none of the answers (A)–(D) is correct

13. One root of \(x^3 + 5x^2 - 13x - 33\) is \(x = 3\). Which of the following is another root?

(A) \(x = -4 + \sqrt{5}\)   (B) \(x = 4 + \sqrt{5}\)   (C) \(x = -3\)   (D) \(x = -8 + 2\sqrt{5}\)

(E) none of the answers (A)–(D) is correct

14. The probability that it is Friday and that a student is absent is 0.03. Since there are 5 school days in a week, the probability that it is Friday is 0.2. What is the probability that a student is absent given that today is Friday?

(A) 3%   (B) 15%   (C) 17%   (D) 20%   (E) none of the answers (A)–(D) is correct
15. Triangle $PAB$ and square $ABCD$ below are in perpendicular planes. Given that $PA = 3$, $PB = 4$, and $AB = 5$, what is $PD$?

(A) 5  (B) $\sqrt{34}$  (C) $\sqrt{41}$  (D) $2\sqrt{13}$  (E) 8

16. Given a cube, how many line segments can be formed whose endpoints are vertices of the cube?

(A) 12  (B) 15  (C) 24  (D) 28  (E) 56

17. Suppose that $a$ and $b$ are nonzero real numbers, and that the equation $x^2 + ax + b = 0$ has solutions $a$ and $b$. Then the pair $(a, b)$ is:

(A) $(-2, 1)$  (B) $(-1, 2)$  (C) $(1, -2)$  (D) $(2, -1)$

(E) none of the answers (A)–(D) is correct

18. The sum of the lengths of the three sides of a right triangle is 56. The sum of the squares of the lengths of the three sides of the same right triangle is 1,250. What is the area of the triangle?

(A) 25  (B) 84  (C) 168  (D) 625  (E) none of the answers (A)–(D) is correct

19. For how many positive integers $n$ is $n^2 - 3n + 2$ a prime number?

(A) none  (B) one  (C) two  (D) more than two, but finitely many  (E) infinitely many

20. Suppose July of year $N$ has five Mondays. Which of the following must occur five times in the August of year $N$? (Note: Both months have 31 days.)

(A) Monday  (B) Tuesday  (C) Wednesday  (D) Thursday  (E) Friday
21. Four distinct circles are drawn in a plane. What is the maximum number of points where at least two of the circles intersect?

(A) 8 (B) 9 (C) 10 (D) 12 (E) 16

22. Let $a$, $b$, and $c$ be real numbers such that $a - 7b + 8c = 4$ and $8a + 4b - c = 7$. Then $a^2 - b^2 + c^2$ equals:

(A) 0 (B) 1 (C) 4 (D) 7 (E) 8

23. I want to increase a cylindrical can to have five times the original volume. Due to package limitations, I can only increase the height of the can by 25%. Assuming I do this, by what percentage must the radius of the can be increased to make the volume five times as large.

(A) 100% (B) 150% (C) 200% (D) 250% (E) none of the answers (A)–(D) is correct

24. Which of the following expressions is equivalent to \[
\frac{6x}{x^2 + 9} = \frac{6x}{x^2 + 9}
\]?

(A) \(\frac{6x}{x^2 + 18}\) (B) \(\frac{6x}{x^2 + 9}\) (C) \(\frac{9}{x^2 + 9}\) (D) \(\frac{1}{x^2}\) (E) none of the answers (A)–(D) is correct

25. The formula $A = P \left(1 + \frac{r}{n}\right)^{nt}$ calculates interest given principal $P$, rate $r$, number $n$ of compounds per year, and time $t$ in years. How long will it take money to double if it is invested at 6% compounded monthly?

(A) \(\frac{\ln 2}{12 \ln 1.5}\) years (B) \(\frac{12 \ln 1.5}{\ln 2}\) years (C) \(\frac{\ln 2}{12 \ln 1.005}\) years (D) \(\frac{12 \ln 1.005}{\ln 2}\) years (E) none of the answers (A)–(D) is correct

26. Which of the following is equivalent to the expression $\sqrt{3 + 2\sqrt{2}}$?

(A) $17 + 12\sqrt{2}$ (B) $1 + \sqrt{2}$ (C) $3^{1/2} + 2^{3/4}$ (D) $121$ (E) none of the answers (A)–(D) is correct
27. Solve the equation $25^{2x-1} = 125^{x+4}$.

(A) $\frac{4}{9}$  (B) $\frac{5}{14}$  (C) $\frac{9}{4}$  (D) $\frac{14}{5}$  (E) none of the answers (A)–(D) is correct

28. Consider the two statements:

I. Some Mems are not Ens.

II. No Ens are Veens.

If “some” means “at least one”, we can conclude that:

(A) Some Mems are not Veens.  (B) Some Veens are not Mems.

(C) No Mem is a Veen.  (D) Some Mems are Veens.

(E) None of the answers (A)–(D) is deducible from the given statements.

29. Square $ABCD$ has side length 10 units, point $E$ is on the line segment $BC$, and the area of $\triangle ABE$ is 40 square units. What is the length of line segment $BE$?

(A) 4 units  (B) 5 units  (C) 6 units  (D) 7 units  (E) 8 units

30. A softball team played ten games, scoring 1, 2, 3, 4, 5, 6, 7, 8, 9, and 10 runs. They lost by one run in exactly five games. In each of the other games, they scored twice as many runs as their opponent. How many total runs did their opponents score?

(A) 35  (B) 40  (C) 45  (D) 50  (E) 55