2018
High School Math Contest

Level 3
Exam
Key

Lenoir-Rhyne University
Donald and Helen Schort School of
Mathematics and Computing Sciences

This exam was prepared by Stephanie Hays, Timothy Goldberg, and Sarah Nelson of LRU.
1. The average number of vehicles waiting in line at a toll booth of a super highway is modeled by the function

\[ n(x) = \frac{x^2}{0.5(1 - x)}. \]

where \( x \) is the quantity between 0 and 1 known as the traffic intensity. To the nearest tenth, find the average number of vehicles waiting if the traffic intensity is 0.81.

(A) 8.5 vehicles  (B) **6.9 vehicles**  (C) 3.5 vehicles  (D) 1.6 vehicles

(E) None of the answers (A)–(D) is correct.

2. A jar containing only nickels and dimes contains a total of 70 coins. The value of all the coins in the jar is $5.30. How many nickels and dimes are in the jar?

(A) 35 nickels and 36 dimes  (B) 36 nickels and 34 dimes  (C) **34 nickels and 36 dimes**

(D) 33 nickels and 37 dimes  (E) None of the answers (A)–(D) is correct.

3. Solve the inequality \(|x + 7| + 5 \leq 7.\)

(A) \((-\infty, -9] \cup [-5, \infty)\)  (B) \([-9, 7]\)  (C) \([-9, -5]\)

(D) \((-9, 5)\)  (E) None of the answers (A)–(D) is correct.

4. You stop for lunch at a local pizza shop where each pizza is cut into 8 slices. Would your hunger be better satisfied with one slice from a 16-inch diameter pizza or two slices from a 12-inch diameter pizza?

(A) one slice from a 16-inch diameter pizza  (B) **two slices from a 12-inch diameter pizza**

(C) Both options would equally satisfy your hunger.  (D) It is impossible to determine.

(E) None of the answers (A)–(D) is correct.

5. Let \( n \) be the number of ordered pairs \((x, y)\) which satisfy \(4y - 3x = 12\) and \(x^2 + y^2 \leq 25\). Then \( n \) is:

(A) zero  (B) one  (C) two  (D) **more than two**

(E) None of the answers (A)–(D) is correct.
6. The incenter of a triangle is:

(A) the point where the three angle bisectors intersect

(B) the point where the three medians intersect

(C) the point where the three perpendicular bisectors intersect

(D) the point where the three altitudes intersect

(E) None of the answers (A)–(D) is correct.

7. The radius of the Earth at the equator is approximately 4000 miles. Suppose a jet plane flies once around Earth at a speed of 500 miles per hour relative to Earth. If the flight path is a negligible height above the equator, then the number of hours of flight is approximately:

(A) 8  (B) 25  (C) 50  (D) 75  (E) None of the answers (A)–(D) is correct.

8. Emma will roll two standard dice labeled 1 through 6. She will record the sum of the two numbers after each roll. She will roll the two dice 540 times. How many times should Emma expect the sum to equal 5?

(A) 4  (B) 9  (C) 50  (D) 480  (E) None of the answers (A)–(D) is correct.

9. Which of the following expressions represents the amount of money invested at 5% compounded continuously for 3 years that will yield $820?

(A) \( 820 e^{15} \)

(B) \( 820 e^{0.15} \)

(C) \( \frac{820}{e^{15}} \)

(D) \( \frac{820}{e^{0.15}} \)

(E) None of the answers (A)–(D) is correct.

10. Consider the following statements:

I. \( \sqrt{-3} \sqrt{-12} = \sqrt{(-3)(-12)} \)

II. \( \sqrt{(-3)(-12)} = \sqrt{36} \)

III. \( \sqrt{36} = 6 \)

Of these, which are incorrect?

(A) I only  (B) II only  (C) III only  (D) I and III only  (E) None

11. Each edge of a cube is colored either red or black. Every face of the cube has at least one black edge. The smallest number of black edges possible is:

(A) 2  (B) 3  (C) 4  (D) 5  (E) None of the answers (A)–(D) is correct.
12. The number of points with positive rational coordinates selected from the set of points in the \(xy\)-plane such that \(x + y \leq 7\) is:

(A) 9  (B) 10  (C) 14  (D) infinite  (E) None of the answers (A)–(D) is correct.

13. Find the annual percent increase or decrease modeled by the equation \(y = 0.35(2.3)^x\).

(A) 230\% increase  (B) 130\% increase  (C) 65\% decrease  (D) 30\% decrease  (E) None of the answers (A)–(D) is correct.

14. Which of the following is equivalent to the equation \(4^2 = x\)?

(A) \(\log_x 4 = 2\)  (B) \(\log_4 2 = x\)  (C) \(\log_2 x = 4\)  (D) \(\log_4 x = 2\)  (E) None of the answers (A)–(D) is correct.

15. How many values of \(x\) satisfy the equation \(\frac{5x}{x - 3} - 7 = \frac{15}{x - 3}\)?

(A) zero  (B) one  (C) two  (D) three  (E) None of the answers (A)–(D) is correct.

16. Circle 1 is circumscribed about a given square, and circle 2 is inscribed in the square. What is the ratio of the area of circle 1 to the area of circle 2?

(A) \(\sqrt{2}\)  (B) 2  (C) \(\sqrt{3}\)  (D) \(2\sqrt{3}\)  (E) None of the answers (A)–(D) is correct.

17. Five cards are lying on a table as shown.

Each card has a letter on one side and a whole number on the other side. John said, “If a card has a vowel on one side, then it has an even number on the other side.” Stephanie showed John was wrong by turning over just one card. Which card did Stephanie turn over?

(A) 3  (B) P  (C) 4  (D) Q  (E) 6

18. Michael is 25 years younger than his father. The sum of their ages is 53. How old is Michael?

(A) 14  (B) 25  (C) 28  (D) 39  (E) None of the answers (A)–(D) is correct.
19. If
\[ f(x) = \begin{cases} 
3x + 1 & \text{if } x < -1 \\
-2x - 5 & \text{if } x \geq -1 
\end{cases} \]
then what is \( f(2) \)?
(A) \(-9\)  (B) \(-8\)  (C) \(-3\)  (D) \(1\)  (E) None of the answers (A)–(D) is correct.

20. Use the figure below to determine the exact value of \( \sin \theta \).

![Diagram](image)

(A) \(\frac{12}{13}\)  (B) \(-\frac{5}{13}\)  (C) \(-\frac{5}{12}\)  (D) \(1\)  (E) None of the answers (A)–(D) is correct.

21. Suppose the graph of \( y = |x| \) is vertically stretched by a factor of 2.8, then reflected across the \( x \)-axis, and then shifted 0.78 units downward. What is an equation for the resulting graph?
(A) \( y = -2.8|x| - 0.78 \)  (B) \( y = 2.8|x| - 0.78 \)  (C) \( y = 2.8|x - 0.78| \)  (D) \( y = 2.8|x| - 0.78 \)  (E) None of the answers (A)–(D) is correct.

22. In parallelogram \( FGHI \), diagonals \( IG \) and \( FH \) are drawn to intersect at point \( M \). Which of the following statements must be true?
(A) \( \triangle FGI \) must be an obtuse triangle.
(B) \( \triangle HGI \) must be an acute triangle.
(C) \( \triangle FMG \cong \triangle HMG \).
(D) \( \triangle GMH \cong \triangle IMF \).
(E) None of the answers (A)–(D) is correct.

23. The graph of the function \( f(x) = 2x^4 + 4x^3 - 5x^2 - 5x + 6 \) is given below.
How many zeros of the function must be complex (not real)?
(A) 0  (B) 1  (C) 2  (D) 3  (E) None of the answers (A)–(D) is correct.

24. How many three-digit integers are exactly 911 more than a two-digit integer?
(A) 11  (B) 79  (C) 89  (D) 90  (E) None of the answers (A)–(D) is correct.

25. Which of the following expressions is exactly equivalent to \( \frac{4p - 4}{p} \div \frac{9p - 9}{4p^2} \)?
(A) \( \frac{16p}{9} \)  (B) \( \frac{16p^3 - 16p^2}{9p^2 - 9p} \)  (C) \( \frac{9}{16p} \)  (D) \( \frac{36p^2 + 72p + 36}{4p^3} \)
(E) None of the answers (A)–(D) is correct.

26. It takes David 2 hours to mow his lawn. It takes Elizabeth 3 hours to mow the same lawn. At the same pace, how long would it take them to mow the lawn if they do the job together?
(A) 2 \( \frac{1}{2} \) hours  (B) 1 \( \frac{1}{5} \) hours  (C) 1 \( \frac{1}{6} \) hours  (D) \( \frac{5}{6} \) hours
(E) None of the answers (A)–(D) is correct.

27. A boat costs $15,500 and decreases in value by 10% per year. How much will the boat be worth in 5 years?
(A) $15,450  (B) $9,153  (C) $8,237  (D) $155
(E) None of the answers (A)–(D) is correct.

28. A recycling box holds 151,875 cm\(^3\) of material. The base of the box is a square with side length 45 cm. What is the height of the box?
(A) 33.8 cm  (B) 75 cm  (C) 1687.5 cm  (D) 3375 cm  (E) None of the answers (A)–(D) is correct.
29. How many revolutions did the Earth make around its axis in the 20th century?
(The 20th century was a century that began on January 1, 1901 and ended on December 31, 2000. It was the tenth and final century of the 2nd millennium.)

(A) 36,500  (B) 36,525  (C) 36,550  (D) 36,575

(E) None of the answers (A)–(D) is correct.

30. Which of the following expressions is equivalent to log₃ 2 + 2 log₃ 2?

(A) log₃ 8  (B) log₈ 3  (C) log₃ 4  (D) 1  (E) ln 8