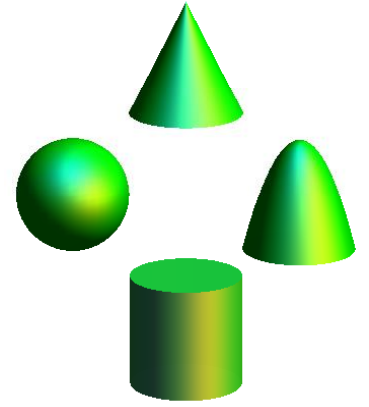




2019
High School Math Contest

Level 3
Exam



Lenoir-Rhyne University
*Donald and Helen Schort School of
Mathematics and Computing Sciences*

This exam was prepared by Stephanie Hays and Timothy Goldberg of LRU.

Do's and Don'ts:

- **Do NOT** open this booklet until you are instructed to do so.
- **NO Calculators** (or other electronic devices)
- Contestants with electronic devices (on or off) will be **disqualified!**
And their **team** will be **disqualified!**
(The other team members will continue to participate in the individual contest.)

DIRECTIONS:

- **The Score Card:**
Write:
 - Your **NAME** on the “name line” (of course).
 - “**Level 3**” on the “subject line”.
 - Your **SCHOOL** on the “**Date Line**”
- **Clearly mark** ONE bubble using **#2 PENCIL**.
 - Light marks will be counted as unmarked!
 - **Completely erase** any changes.
- You **can** write on this test booklet (but the test booklet will not be graded).
- **Tie Breakers:** In case of ties, the person with the least number of *wrong answers* wins.
(A *blank* is better than *incorrect!*)
- **The Exam:** 30 problems, 70 minutes.
- Problems are NOT in order of increasing difficulty, so feel free to skip around. (Just be careful to mark your answers correctly on your score card!)

- **WAIT** for the signal to begin.

1. Which of the following is equivalent to $(x - 3)^2 - (x^2 + 7x - 11) + 2^5$?

(A) $2x^2 - 7x + 52$ (B) $x + 30$ (C) $12 - 7x$ (D) $52 - 13x$

(E) None of the answers (A)–(D) is correct.

2. If the function is one-to-one, find an equation for its inverse.

$$f(x) = x^3 - 1$$

(A) $f(x)$ is not a one-to-one function. (B) $f^{-1}(x) = \sqrt[3]{x} + 1$ (C) $f^{-1}(x) = \sqrt[3]{x + 1}$

(D) $f^{-1}(x) = \sqrt[3]{x - 1}$ (E) None of the answers (A)–(D) is correct.

3. There were 6 teachers. Each teacher shook hands once with each other. How many handshakes took place?

(A) 15 (B) 12 (C) 10 (D) 6 (E) None of the answers (A)–(D) is correct.

4. Which of the following are true?

(i) When you double the side length of a square, the perimeter doubles.

(ii) When you double the side length of a square, the area quadruples.

(iii) When you double the side length of a cube, the volume increases by a factor of 8.

(A) Only (i) and (ii) are true. (B) Only (i) and (iii) are true.

(C) Only (ii) and (iii) are true. (D) All are true. (E) None are true.

5. Simplify $\frac{4 + \frac{2}{x}}{\frac{x}{3} + \frac{1}{6}}$.

(A) 1 (B) 12 (C) $\frac{x}{12}$ (D) $\frac{12}{x}$ (E) None of the answers (A)–(D) is correct.

6. Two years ago, Cua invested \$2400 in a savings account earning 2% interest compounded annually. To the nearest dollar, how much money is in her account now?

- (A) \$2,424 (B) \$2,497 (C) \$2,518 (D) \$4,800
(E) None of the answers (A)–(D) is correct.

7. Is the relation described by the values also a function? Justify your answer.

x	y
-5	0
-2	2
0	4
-2	-3
4	-5
7	7
8	11

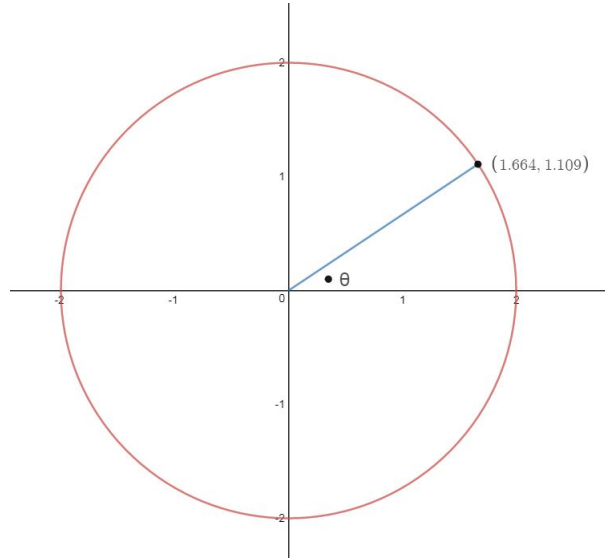
- (A) Yes, because all the output values are different. (B) Yes, because it is not linear.
(C) No, because there is an input value that has two different output values.
(D) No, because it is not linear. (E) None of the answers (A)–(D) is correct.

8. What type of quadrilaterals have two pairs of parallel sides?

- (i) Rectangle
(ii) Rhombus
(iii) Kite
(iv) Parallelogram

- (A) Only (i) (B) Only (i) and (ii) (C) Only (i) and (iii) (D) Only (i) and (iv)
(E) None of the answers (A)–(D) is correct.

9. Below is a circle together with a radius, with the angle between the radius and the positive x -axis labeled θ . (The point's coordinates are rounded to the nearest three decimal places.)
What is the approximate value of $\sin(\theta)$?



- (A) 1.664 (B) 1.109 (C) 0.8320 (D) 0.5545
(E) None of the answers (A)–(D) is correct.
10. A farmer had nineteen sheep, and all but eight died. How many does she have left?
(A) 8 (B) 11 (C) 19 (D) 27 (E) None of the answers (A)–(D) is correct.
11. Which of the following binomials is a factor of $x^3 - 8x^2 + 5x + 14$?
(A) $x - 6$ (B) $x + 3$ (C) $x + 9$ (D) $x - 2$ (E) None of the answers (A)–(D) is correct.
12. How can the graph of $f(x) = (x - 3)^2 - 4$ be obtained from the graph of $y = x^2$?
(A) Shift the graph 3 units left and 4 units down.
(B) Shift the graph 4 units right and 3 units down.
(C) Shift the graph 3 units left and 4 units up.
(D) Shift the graph 3 units right and 4 units down.
(E) None of the answers (A)–(D) is correct.

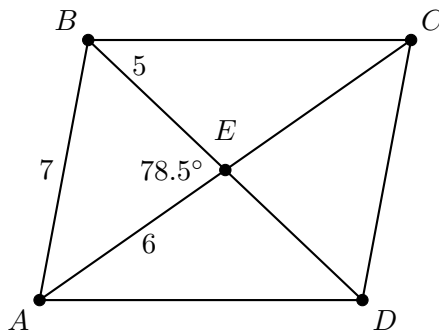
13. Cody went to a local coffee shop and bought five medium coffees and six egg sandwiches for his study group. He was charged \$32.00. Late that semester, Roxy visited the same coffee shop to buy five medium coffees and seven egg sandwiches for the study group. She was charged \$35.25. What is the cost of a medium coffee, and what is the cost of an egg sandwich?
- (A) Medium coffee: \$2.50; Egg sandwich: \$3.25
 (B) Medium coffee: \$3.00; Egg sandwich: \$3.50
 (C) Medium coffee: \$2.00; Egg sandwich: \$3.00
 (D) Medium coffee: \$3.50; Egg sandwich: \$3.00
 (E) None of the answers (A)–(D) is correct.
14. Martha's mother had four children, and she often named them after the days of the week. Her first child was named Monday, her second child was named Tuesday, and her third child was named Wednesday. What was her last child's name?
- (A) Thursday (B) Friday (C) Saturday (D) Sunday
 (E) None of the answers (A)–(D) is correct.
15. A polynomial has roots at $x = -1$ and $x = 3i$. Assuming all coefficients are real, what is the minimum possible degree of the polynomial?
- (A) 6 (B) 5 (C) 4 (D) 3 (E) None of the answers (A)–(D) is correct.

16. Solve for z :

$$xy^{zt} = d$$

- (A) $z = \frac{\log(d/x)}{t \log(y)}$ (B) $z = \frac{\sqrt{d}}{t \sqrt{xy}}$ (C) $z = \frac{\log(d)}{t \log(xy)}$ (D) $z = \log\left(\frac{d}{xy}\right)$
 (E) None of the answers (A)–(D) is correct.

17. If $ABCD$ in the figure below is a parallelogram, what is the length of segment \overline{BD} ?



- (A) 10 (B) 11 (C) 12 (D) 14 (E) None of the answers (A)–(D) is correct.
18. The following table represents the estimates of the percentages of various demographic groups within the population of Catawba County in 2018.

Group	Percentage of Population
White	75.2%
Hispanic or Latino	9.6%
Black	8.7%
Asian	4.2%
Multiracial	1.6%
American Indian and Alaska Native	0.6%
Native Hawaiian or Other Pacific Islander	0.1%

If this data were to be represented as a pie chart, what would be the approximate measure of the central angle for the sector that represents the Black population?

- (A) 8.7° (B) 11.49° (C) 31.32° (D) 36° (E) None of the answers (A)–(D) is correct.
19. What number most likely comes next in the following sequence?

3, 5, 9, 17, 33, ...

- (A) 44 (B) 49 (C) 64 (D) 65 (E) None of the answers (A)–(D) is correct.

20. Find the solutions to the equation

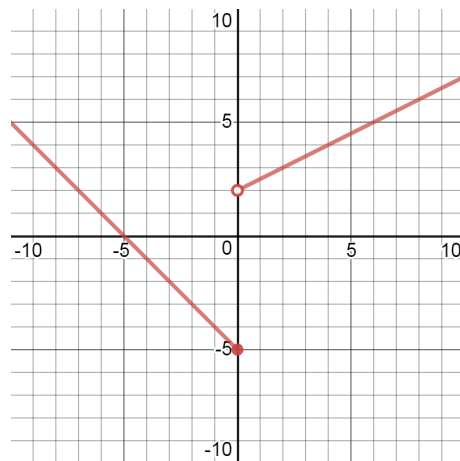
$$\frac{y}{y+4} + \frac{8y+28}{y^2+7y+12} = \frac{4}{y+3}.$$

- (A) 4 and 3 (B) -6 and 5 (C) -3 and -4 (D) There are no solutions.
(E) None of the answers (A)–(D) is correct.

21. A police officer seals off a rectangular area which was the scene of a car collision. She uses a roll of yellow police tape that is 120 feet long. What is the maximum area she can seal with this tape?

- (A) 500 square feet (B) 675 square feet (C) 800 square feet (D) 875 square feet
(E) None of the answers (A)–(D) is correct.

22. Which function is represented by the following graph?



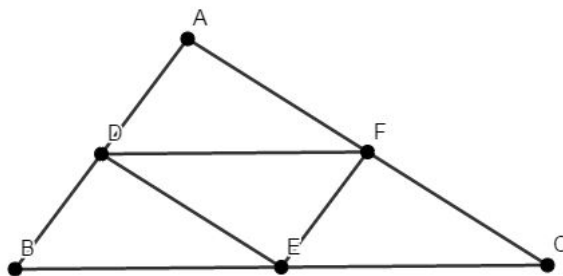
- (A) $f(x) = \begin{cases} -x - 5 & \text{if } x \leq 0, \\ \frac{1}{2}x + 2 & \text{if } x > 0. \end{cases}$ (B) $f(x) = \begin{cases} x - 5 & \text{if } x \leq 0, \\ \frac{1}{2}x + 2 & \text{if } x > 0. \end{cases}$
(C) $f(x) = \begin{cases} -x + 5 & \text{if } x \leq 0, \\ -\frac{1}{2}x + 2 & \text{if } x > 0. \end{cases}$ (D) $f(x) = \begin{cases} x + 5 & \text{if } x \leq 0, \\ -\frac{1}{2}x + 2 & \text{if } x > 0. \end{cases}$
(E) None of the answers (A)–(D) is correct.

23. Suppose that we found that a 95% confidence interval for the mean number of hours that high school students used their cell phones per day was 2.53 hours to 3.71 hours. Which of the following is a correct interpretation of this result?
- (A) We can be confident that 95% of all high school students use their cell phones between 2.53 hours and 3.71 hours per day.
- (B) The probability is 0.95 that the mean number of hours that high school students use their cell phones per day is between 2.53 hours and 3.71 hours.
- (C) 95% of the time, a high school student will use their cell phone between 2.53 hours and 3.71 hours per day.
- (D) We are 95% confident that the true mean amount of time that high school students spend using their cell phones per day is between 2.53 hours and 3.71 hours.
- (E) None of the answers (A)–(D) is correct.

24. Simplify the expression $8 \left(\frac{4x^5y}{16xy^4} \right)^3$.
- (A) $\frac{x^4}{4y^3}$ (B) $\frac{2x^4}{y^3}$ (C) $\frac{x^{12}}{64y^9}$ (D) $\frac{x^{12}}{8y^9}$ (E) None of the answers (A)–(D) is correct.

25. Which three shapes can be created as a cross section of a cube?
- (A) square, octagon, and equilateral triangle
- (B) scalene triangle, heptagon, and rectangle
- (C) equilateral triangle, square, and pentagon
- (D) equilateral triangle, decagon, and hexagon
- (E) None of the answers (A)–(D) is correct.

26. Suppose that the segments \overline{AD} and \overline{BD} have the same length, and the segments \overline{DF} and \overline{BE} are parallel and have the same length. Which of the points D , E , and F *must* be a midpoint of one of the edges of triangle ABC ?

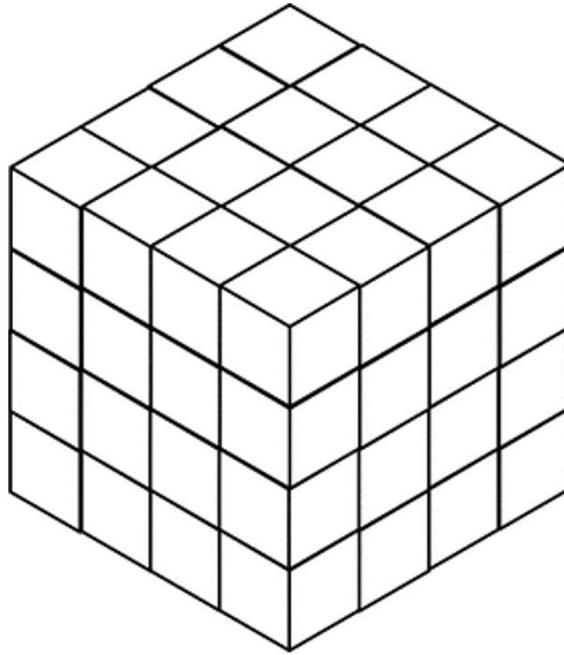


- (A) D and E only (B) E and F only (C) D and F only (D) D , E , and F
 (E) None of the answers (A)–(D) is correct.

27. Factor the expression

$$3x^3 - x^2y + 6x^2y - 2xy^2 + 3xy^2 - y^3.$$

- (A) $(3x - 2y)(x + y)$ (B) $(3x - y)(x + y)(x - y)$ (C) $(3x - y)(x + y)^2$ (D) $(3x - y)(x^2 + y^2)$
 (E) None of the answers (A)–(D) is correct.
28. The following object is composed of $4 \times 4 \times 4 = 64$ cubes. How many of these cubes share exactly four faces with other cubes?



- (A) 8 (B) 12 (C) 16 (D) 24 (E) None of the answers (A)–(D) is correct.
29. Solve the following equation:

$$\frac{6}{r} + \frac{12}{r^2 - 4r} = -\frac{2}{4 - r}$$

- (A) $r = 0$ (B) $r = 3$ (C) $r = 4$ (D) There is no solution.
 (E) None of the answers (A)–(D) is correct.

