Geometry Exam
Lenoir-Rhyne University
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Do’s and Don’ts:
- Do NOT open this booklet until you are instructed.
- Only approved calculators, such as the TI-83, TI-84, and comparable calculators, are allowed.
- Calculators with “typewriter” keyboards are not permitted.
- Electronic devices (phones, Blackberries, etc), other than approved calculators, are not permitted during the exam. (Leave these with your supervisor.)
- Contestants with forbidden electronic devices (on or off) will be disqualified! And their team will be disqualified!
   (The other team members will continue to participate in the individual contest.)

DIRECTIONS:
- The Score Card:
  - For the GEOMETRY exam, you must use the score card highlighted in GREEN.
  - Write “GEOMETRY” on the subject line.
  - Fill out your name.
  - Fill in your SCHOOL name.
- Clearly mark ONE bubble using #2 PENCIL.
  - Light marks will be counted as unmarked!
  - Completely erase any changes.
- You can write on this test booklet. (But the test booklet will not be graded.)
- Tie Breakers: In case of ties, the person with the least number of wrong answers wins.
  (Thus, a blank is better than incorrect!)
- The Exam: 40 problems, 70 minutes.
- WAIT for the signal to begin.
1. One of the interior angles of a triangle is $30^\circ$ and one of the exterior angles of the triangle is $115^\circ$. What is the measure of the largest interior angle of the triangle?

a) $65^\circ$.
b) $75^\circ$.
c) $85^\circ$.
d) $95^\circ$.
e) Cannot be determined.

2. Two of the vertices of a triangle are $(3, 5)$ and $(2, 1)$. If the triangle is isosceles, which of the following could be the third vertex of the triangle?

a) $(0, 4)$.
b) $(4, 5)$.
c) $(2, 5)$.
d) $(4, 0)$.
e) $(4, 1)$.

3. The length of the hypotenuse of a right triangle is 5 inches and the measure of one of the actual angles of the triangle is $60^\circ$. Which of the following is the length of one leg of the triangle?

a) $\frac{5}{3}$ inches.
b) $\frac{5}{2}$ inches.
c) $5\sqrt{3}$ inches.
d) 3 inches.
e) $\frac{9}{2}$ inches.

4. Circle 1 has a circumference of $100\pi$ centimeters and Circle 2 has a circumference of $150\pi$ centimeters. The length of the diameter of Circle 2 exceeds the length of the diameter of Circle 1 by:

a) 25 cm.
b) 50 cm.
c) 75 cm.
d) 100 cm.
e) 200 cm.
5. Find the area of the parallelogram $ABCD$ shown below if the measure of segments $AB$, $BC$, and $DE$ are 6 units, 2 units, and 1 unit respectively and $\angle AED$ is a right angle.

a) 5 square units.
b) 12 square units.
c) $5\sqrt{3}$ square units.
d) $6\sqrt{3}$ square units.
e) 16 square units.

6. A square has vertices (-1, -1), (-1, 3), (3, -1), (3, 3) and a circle is inscribed in this square. Find the center and radius of this inscribed circle.

a) Center: (0, 0); Radius: 1 unit.
b) Center: (0, 0); Radius: 2 units.
c) Center: (1, 1); Radius: 2 units.
d) Center: (1, 1); Radius: 1 unit.
e) Center: (2, 2); Radius: 1 unit.

7. Find the area of a triangle whose vertices are (-2, -3), (1, 5), and (5, -3).

a) 12 square units.
b) 20 square units.
c) 24 square units.
d) 28 square units.
e) 56 square units.

8. If the interior angle of a regular polygon is greater than $100^\circ$, then which of the following statements is true?

a) The polygon has exactly 4 sides.
b) The polygon has at most 4 sides.
c) The polygon has exactly 5 sides.
d) The polygon has at most 5 sides.
e) The polygon has at least 5 sides.
9. In the figure, lines \(m\) and \(n\) are parallel, the measure of angle 2 is 10\(x\), and the measure of angle 3 is 8\(x\). Determine the measure of angle 1.

a) 95\(^\circ\).
b) 100\(^\circ\).
c) 105\(^\circ\).
d) 110\(^\circ\).
e) 120\(^\circ\).

10. The diameter of circle \(A\) is twice the length of the diameter of circle \(B\). How are the circumferences of the two circles related?

a) The circumference of circle \(A\) is twice the circumference of circle \(B\).
b) The circumference of circle \(B\) is twice the circumference of circle \(A\).
c) The circumference of circle \(A\) is four times the circumference of circle \(B\).
d) The circumference of circle \(B\) is four times the circumference of circle \(A\).
e) The circumference of circle \(A\) is 2\(\pi\) units more than the circumference of circle \(B\).

11. A city wants to locate a hospital the same distance from three different fire stations. The fire stations are located at the vertices of a large triangle. Which of the following points is the best location for the hospital?

a) The intersection of the three medians of the triangle.
b) The intersection of the three altitudes of the triangle.
c) The intersection of the two midlines of the triangle.
d) The intersection of the three angle bisectors of the triangle.
e) The intersection of the three perpendicular bisectors of the sides of the triangle.

12. Construct a segment and mark its midpoint. Now construct a second segment that is perpendicular to the first segment at the midpoint, is not bisected by the first segment, and has endpoints that are not on the first segment. The endpoints of the two segments are the vertices of a convex quadrilateral. Which of the following best describes this quadrilateral?

a) It is a rectangle.
b) It is a rhombus.
c) It is a parallelogram that is neither a rhombus nor a rectangle.
d) It is a kite.
e) It is a trapezoid.
13. In the figure, $BC$ is the base of isosceles triangle $ABC$. $BC$ is a diameter of the circle, point $A$ is on the circle, $BC \parallel DE$, and $DE$ is tangent to the circle. What is the ratio of the area of triangle $ABC$ to the area of trapezoid $BCED$?

a) 1:4.

b) 1:5.

c) 2:5.

d) 1:3.

e) 3:4.

14. If the line through the points $(0, 3)$ and $(3, 7)$ is perpendicular to the line through the points $(3, 5)$ and $(7, k)$, then the value of $k$ is:

a) 2.

b) 8.

c) $\frac{31}{3}$.

d) -1.

e) 1.

15. Which of the following has the largest area?

a) A circle with a radius of 1 unit.

b) A square with a side of 1 unit.

c) A square with a diagonal of 2 units.

d) A regular hexagon with longest diagonal of 2 units.

e) A circle with a circumference of 2 units.

16. Construct a segment and mark its midpoint. Now construct a second segment that is perpendicular to the first segment at the midpoint, is bisected by the first segment, and is congruent to the first segment. The endpoints of the two segments are the vertices of a quadrilateral. Which of the following best describes this quadrilateral?

a) It is a square.

b) It is a rhombus that is not a square.

c) It is a rectangle that is not a square.

d) It is a parallelogram that is neither a rhombus nor a square.

e) It is a trapezoid.
17. A vertical wall 20 feet high casts a shadow 8 feet wide on level ground. If Alex is 5 feet, 5 inches tall, how far away from the wall can he stand and still be entirely in the shade?

   a) 2 feet, 4 inches.
   b) 3 feet, 9 inches.
   c) 4 feet, 3 inches.
   d) 5 feet, 10 inches.
   e) 6 feet, 2 inches.

18. Quadrilateral $ABCD$ is congruent to quadrilateral $BCDA$. What else can you say about quadrilateral $ABCD$?

   a) It is a parallelogram that is not a rectangle nor a rhombus.
   b) It is a rectangle that is not a square.
   c) It is a square.
   d) It is rhombus that is not a square.
   e) No conclusions can be reached concerning quadrilateral $ABCD$.

19. Which of the following is not a platonic solid?

   a) Tetrahedron.
   b) Pentahedron.
   c) Octahedron.
   d) Dodecahedron.
   e) Icosahedron.

20. A closed, right, circular cylinder has a radius of one unit and a height of three units. Its total surface are is:

   a) $4\pi$ square units.
   b) $6\pi$ square units.
   c) $8\pi$ square units.
   d) $3\pi^2$ square units.
   e) $6\pi^2$ square units.
21. In the figure shown below, $AB \parallel CD$, $AB \perp BC$, the measure of $BC$ is 5 units, the measure of $AD$ is 8 units, and the perimeter of quadrilateral $ABCD$ is 37 units. What is the area of quadrilateral $ABCD$?

a) 6.5 square units.
b) 12 square units.
c) 60 square units.
d) 120 square units.
e) 370 square units.

22. $ABCD$ is a square, $E$ is the midpoint of $AB$, and the measure of $DE$ is $\sqrt{5}$ units. What is the area of square $ABCD$?

a) 1 square unit.
b) 2 square units.
c) 4 square units.
d) 5 square units.
e) 6 square units.

23. A sphere has radius one meter. In terms of numerical values, which of the following is largest?

a) The surface area of the sphere in square meters.
b) The volume of the sphere in cubic meters.
c) The radius of the sphere in meters.
d) The diameter of the sphere in meters.
e) The circumference of a great circle of the sphere in meters.

24. Which of the following statements about two non-obtuse angles cannot be true?

a) The angles are complementary.
b) The angles are supplementary.
c) The angles are two of the interior angles of an obtuse triangle.
d) The angles are two of the interior angles of a parallelogram.
e) The angles are the exterior angles at two different vertices of a triangle.
25. The length of the diagonal of a computer monitor is 19 inches and the height and width of the monitor are in a 3:4 ratio. The length of the diagonal of another monitor is 24 inches and the height and width of the second monitor are in a 10:16 ratio. To the nearest tenth of an inch, how much wider is the second monitor than the first?

a) 1.3 inches.
b) 3.8 inches.
c) 4.1 inches.
d) 5.2 inches.
e) 7.3 inches.

26. In the figure, $O$ is the center of the circle, points $A$, $B$, $C$ are on the circle, $m\angle BAC = 30^\circ$, and the radius of the circle is 8 units. What is the area of triangle $ABC$?

![Triangle ABC](image)

a) 48 square units.
b) $16\sqrt{3}$ square units.
c) 24 square units.
d) $32\sqrt{3}$ square units.
e) $64\sqrt{3}$ square units.

27. A square-based pyramid has volume $V$ and height $h$. A right, circular cylinder has volume $V$ and the area of its base is the same as the area of the base of the pyramid. What is the height of the cylinder?

a) $h$.
b) $\frac{1}{3}h$.
c) $3h$.
d) $2h$.
e) $\frac{1}{2}h$.

28. Determine the area of the polygon drawn below on centimeter dot paper.

![Polygon](image)

a) 15 cm$^2$.
b) 16 cm$^2$.
c) 16.5 cm$^2$.
d) 17.5 cm$^2$.
e) 18.5 cm$^2$. 
29. Circles $C_1$ and $C_2$ are coplanar with radii 3 and 8 respectively, and the distance between their centers is 13 (see figure). Let $A$ be a point on $C_1$ and $B$ be a point on $C_2$ such that the line containing $A$ and $B$ is tangent to both circles. What is the length of the segment $AB$?

a) 7.
b) 11.
c) 12.
d) 16.
e) 21.

30. The radii of two coplanar circles are 3 and 6 and the distance between their centers is 12. The length of a common internal tangent is:

a) $3\sqrt{7}$.
b) $2\sqrt{35}$.
c) 8.
d) $\sqrt{135}$.
e) 3.

31. In the diagram below, $O$ is the center of the circle, points $A$, $B$, and $C$ are equally spaced around the circle, the circle has radius 1 unit, and line $m$ is tangent to the circle at point $C$. Think of this circle as a wheel that can be rolled along line $m$. How many times will point $A$ touch line $m$ if the circle is rolled 92 units to the right along line $m$?

a) 12.
b) 13.
c) 14.
d) 15.
e) 16.
32. Four circles of radius $r$ are mutually tangent inside a circle of radius one unit. The radius $r$ is:

a) 1 unit.
b) $\frac{1}{2}$ unit.
c) $\sqrt{2} - 1$ unit.
d) $\frac{1}{4}$ unit.
e) $\frac{\sqrt{5}}{5}$ unit.

33. One of the exterior angles of a regular polygon is $9^\circ$. How many sides does the polygon have?

a) 10.
b) 20.
c) 30.
d) 40.
e) 50.

34. The sum of the measures of all but one of the interior angles in a convex polygon is $1020^\circ$. How many sides does this polygon have?

a) 5.
b) 6.
c) 7.
d) 8.
e) Cannot be determined.

35. The area of the circle inscribed in an equilateral triangle is $27\pi$ square units. What is the area of the triangle?

a) $108\sqrt{3}$ square units.
b) $81\sqrt{3}$ square units.
c) $27\sqrt{3}$ square units.
d) $40.5\pi$ square units.
e) $36\pi$ square units.
36. In the figure, $O$ is the center of the circle, $m\angle AOB = 30^\circ$, and $DA \parallel CB$. If the diameter of the circle is 8 units, what is the area of the shaded region?

a) \( \frac{32}{3} \pi + 8\sqrt{3} \) square units.

b) \( \frac{8}{3} \pi + 8\sqrt{3} \) square units.

c) \( \frac{32}{3} \pi + 4\sqrt{3} \) square units.

d) \( \frac{8}{3} \pi + 4\sqrt{3} \) square units.

e) \( 4\pi + 4\sqrt{3} \) square units.

37. In the figure, $ACEF$ is a square, $AB$ is twice as long as $CD$, and $BF = DF$. Which of the following statements is true about the areas of regions 1, 2, and 3?

a) Area 1 > Area 2.

b) Area 2 > Area 1.

c) Area 1 > Area 3.

d) Area 3 > Area 1.

e) Area 2 = Area 1.

38. In the figure, $m\angle ABC = 30^\circ$, $\angle BAC = 120^\circ$, and $m\overline{AB} = 4$ units. The two congruent circles are tangent to each other and tangent to the sides of the triangle. What is the area of one of these circles?

a) \( (4 - 2\sqrt{3})\pi \) square units.

b) \( \frac{\pi}{9} \) square units.

c) \( \frac{3\pi}{4} \) square units.

d) \( (4\sqrt{3} - 4)\pi \) square units.

e) \( \frac{(2 - \sqrt{3})\pi}{2} \) square units.
39. In Picture 1, one fourth of the square has been shaded. Picture 2 was obtained by shading one fourth of each of the smaller white squares in Picture 1. Picture 3 was obtained by shading one fourth of each of the smaller white squares in Picture 2. If this series of pictures is continued, the proportion, \( p \), of the square that will be shaded that will be shaded in Picture 10 will satisfy which of the following inequalities?

![Picture 1](image1.png) ![Picture 2](image2.png) ![Picture 3](image3.png)

a) \( 0.25 < p < 0.50 \).
b) \( 0.50 < p < 0.65 \).
c) \( 0.65 < p < 0.80 \).
d) \( 0.80 < p < 0.90 \).
e) \( 0.90 < p < 1.00 \).

40. The lengths of the sides of a triangle are all distinct whole numbers. If the perimeter of the triangle is 15 cm, what is the maximum area of the triangle?

a) \( \frac{3}{4} \sqrt{65} \text{ cm}^2 \).
b) \( \frac{15}{4} \sqrt{33} \text{ cm}^2 \).
c) \( \frac{15}{4} \sqrt{3} \text{ cm}^2 \).
d) \( \frac{15}{4} \sqrt{7} \text{ cm}^2 \).
e) \( \frac{25}{4} \sqrt{5} \text{ cm}^2 \).