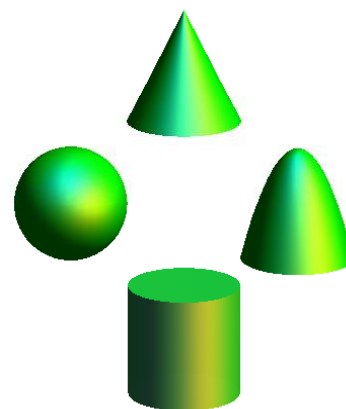




2023
High School Math Contest



Lenoir-Rhyne University
*Donald and Helen Schort School of
Mathematics and Computing Sciences*

This exam was prepared by
Stephanie Hays, CVCC Professor & LR Graduate
Assisted by Dr. Timothy Goldberg, LR Professor.

1. Given that $a^{1/n} = 3$ where a is a real number and n is an integer greater than 1, all of the following are possible values of a **except** which one?

(A) 3 (B) 9 (C) 27 (D) 81 (E) None of the answers (A)–(D) is correct.

2. How many zeros (including complex and counting multiplicity) does the function $h(x) = 7x^2 - 3x^3 + 7 - 12x$ have?

(A) 2 (B) 3 (C) 6 (D) 7 (E) None of the answers (A)–(D) is correct.

3. Consider the function

$$f(x) = \begin{cases} 3x^2 + x - 4 & \text{for } x < -3, \\ -2x + 4 & \text{for } x \geq -3. \end{cases}$$

Find $f(-2)$.

(A) 0 (B) 6 (C) 8 (D) 10 (E) None of the answers (A)–(D) is correct.

4. Let x be the remainder when $n^2 + 10n + 18$ is divided by $n + 5$, and let y be the remainder when $m^2 - 28$ is divided by $m - 5$. What is value of xy ?

(A) 21 (B) -21 (C) -279 (D) 279 (E) None of the answers (A)–(D) is correct.

5. Below is a list of possible zeros for a function.

i. $x = 16$

iv. $x = -4$

ii. $x = -16$

v. $x = \frac{3}{2}$

iii. $x = 4$

vi. $x = -\frac{3}{2}$

Which of the zeros in the list above are zeros of the function $f(x) = (2x - 3)(x + 4)^2$.

(A) i and v only (B) iii and vi only (C) iv and v only (D) iv and vi only

(E) None of the answers (A)–(D) is correct.

6. Riley is 9 years old and her mother is 35 years old. How many years until Riley's mother is 3 times her age?

(A) 4 (B) 5 (C) 7 (D) 13 (E) None of the answers (A)–(D) is correct.

7. If x is any number other than 4 or -8 , what is the most simplified version of the following expression?

$$\frac{x^2 + 12x + 32}{x + 8} \div \frac{x^2 - 16}{2x - 8}.$$

(A) 2 (B) $\frac{1}{2}$ (C) $\frac{2x-8}{x+8}$ (D) $\frac{(x+4)^2}{2}$ (E) None of the answers (A)–(D) is correct.

8. The period, T , of a simple pendulum is the time it take to complete a full swing back-and-forth. The factors that influence the period are L , the length of the pendulum, and g , the acceleration due to gravity, and these quantities are related by the equation

$$T = 2\pi\sqrt{\frac{L}{g}}.$$

If the acceleration due to gravity is 9.8 meters per seconds squared and the period of a simple pendulum is 3 seconds, what is the length of the pendulum?

(A) $2\pi\sqrt{\frac{3}{9.8}}$ meters (B) $9.8\left(\frac{3}{2\pi}\right)^2$ meters (C) $(3 - 2\pi)^2(9.8)$ meters

(D) $\frac{3}{2\pi}\sqrt{9.8}$ meters (E) None of the answers (A)–(D) is correct.

9. Simplify the expression $\left(\frac{3}{x+4}\right) - \left(\frac{5x-7}{x^2-16}\right)$.

(A) $\frac{-2x-5}{x^2-16}$ (B) $\frac{-2x-19}{x^2-16}$ (C) $\frac{8x-19}{x^2-16}$ (D) $\frac{-5x+4}{x^2-16}$

(E) None of the answers (A)–(D) is correct.

10. Solve the equation $125 = 25^{x+5}$.

(A) $\frac{7}{2}$ (B) $-\frac{7}{2}$ (C) -3 (D) 0 (E) None of the answers (A)–(D) is correct.

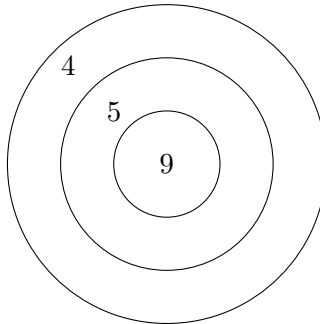
11. Suppose x and y are solutions to the following equations.

$$\begin{aligned}3^{x-y} &= 9 \\3^{x+y} &= 27\end{aligned}$$

What is the value of $x + 3y$?

- (A) 3 **(B) 4** (C) 9 (D) 15 (E) None of the answers (A)–(D) is correct.

12. Liam had six shots at the target shown below.



(Assume if he hits a line, it counts as the higher score.) Which of the following scores could Liam not have gotten?

- (A) 43** (B) 44 (C) 45 (D) 46 (E) None of the answers (A)–(D) is correct.

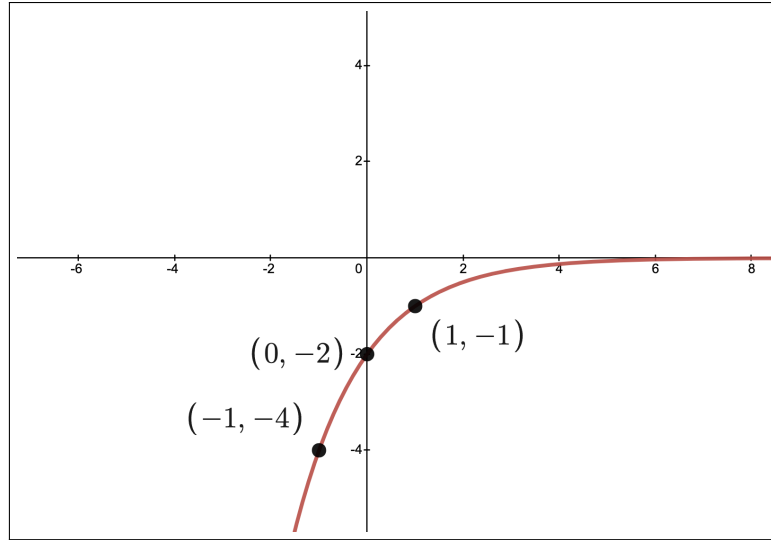
13. A pilot flies her plane with the wind for 2 hours to make a 600 mile trip. Her return flight is against the wind and required 3 hours. Assuming the average wind speed and the average plane air speed each remained constant on both flights, what was the average wind speed?

- (A) 25 mph **(B) 50 mph** (C) 75 mph (D) 100 mph
(E) None of the answers (A)–(D) is correct.

14. What is the amplitude of the function $f(x) = 4\sin(2x) + 3$?

- (A) 2 (B) π **(C) 4** (D) 8 (E) None of the answers (A)–(D) is correct.

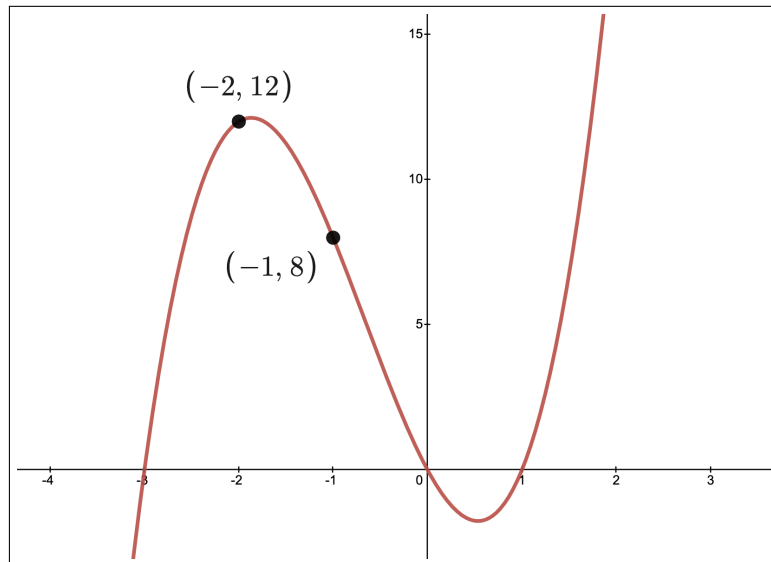
15. Below is the graph of a function of the form $f(x) = a \cdot b^x$.



What is the value of $b - a$?

- (A) -2.5 (B) -1 (C) 0 (D) 2.5 (E) None of the answers (A)–(D) is correct.

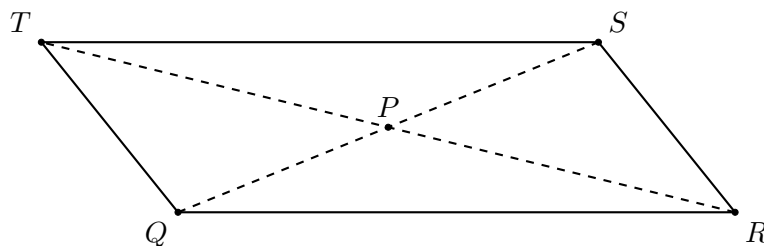
16. Below is the graph of a polynomial, including all of its zeros.



Assuming no multiple roots, what is the leading coefficient of the polynomial?

- (A) 1 (B) 2 (C) 3 (D) 4 (E) None of the answers (A)–(D) is correct.

20. The parallelogram $QRST$ is given below with its diagonals intersecting at point P .



If the length of \overline{RP} is $3(x + 8)$, the length of \overline{QP} is $2(x + 7)$, and the length of \overline{SP} is $4(x - 3)$, find the value of x .

- (A) -10 (B) 1 (C) **13** (D) 36 (E) None of the answers (A)–(D) is correct.
21. Consider the sequence
- $$16, 18, 9, 12, 4, 8, 2, \dots$$
- Choose the best answer for what number comes next, following the pattern given.
- (A) 4 (B) 5 (C) 6 (D) **7** (E) There is no pattern.
22. Suppose a sector of a circle with central angle 4.5 radians has arc length 18 mm. What is the diameter of the circle?
- (A) $\frac{1}{4}$ mm (B) 4 mm (C) **8 mm** (D) 18 mm
- (E) None of the answers (A)–(D) is correct.

23. What is the equation of a circle with center $(3, -4)$ and diameter 6 ?

(A) $(x + 3)^2 + (y - 4)^2 = 36$ (B) $(x - 3)^2 + (y + 4)^2 = 36$ (C) $(x + 3)^2 + (y - 4)^2 = 9$

(D) **$(x - 3)^2 + (y + 4)^2 = 9$** (E) None of the answers (A)–(D) is correct.

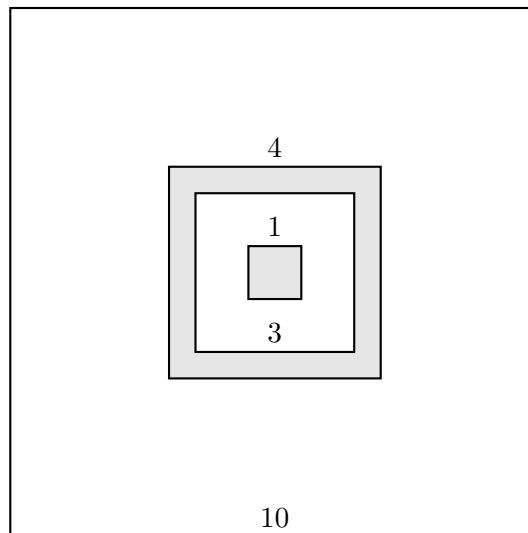
24. The Town of Neighborly Way has a cylindrical water tower with a radius of 16 meters and a height of 8 meters. The Town of Friendly Folks has a cylindrical water tower with a diameter of 8 meters and a height of 16 meters. How many times larger is the volume of Neighborly Way's water tower than that of Friendly Folks'?

- (A) 2 times larger (B) 4 times larger **(C) 8 times larger** (D) 16 times larger
(E) None of the answers (A)–(D) is correct.

25. Suppose that if the measure of one side of a square is increased by 2 centimeters and the measure of the adjacent side is decreased by 2 centimeters, the area of the resulting rectangle is 21 square centimeters. Find the measure of one side of the original square.

- (A) 5** (B) 6 (C) 6.5 (D) 3 (E) None of the answers (A)–(D) is correct.

26. A car dealership is advertising that you can get 50% off any car on the lot if you can throw a dart and land it in a grey area of the square dart board shown below (with the side lengths of the squares labeled). You do not have much experience throwing darts so you make a truly random throw. Given that the dart lands on the board, and not on a line, what is the chance that you get the discount?



- (A) 4% (B) 7% **(C) 8%** (D) 16% (E) None of the answers (A)–(D) is correct.

27. A new dance has come to any room with a tile floor. The dancer starts on a tile. The first move is going forward two tiles. The second move is going back one tile. The dancer repeats these two moves over and over — forward two, back one, forward two, back one, etc. How many moves will it take for the dancer to move forward 10 tiles from the starting tile?

- (A) 17 moves (B) 18 moves (C) 19 moves (D) 20 moves
 (E) None of the answers (A)–(D) is correct.

28. Consider the following expressions.

- i. $(a\sqrt{3})^2$
 ii. $\left(\frac{a^5 \cdot a^7}{a^4}\right)^0 - \frac{(a^4)^4}{a^{16}} + a^3$
 iii. $a^3 \left(1 - a^{\frac{1}{3}}\right) + a^3 \left(a^{\frac{1}{6}}\right)^2$

Which of these expressions are equivalent?

- (A) i and ii only (B) i and iii only (C) ii and iii only (D) i, ii, and iii
 (E) none are equivalent

29. Below is a list of possible zeros for a function.

- | | |
|---------------|--------------|
| i. $x = -4$ | iv. $x = 2$ |
| ii. $x = 4$ | v. $x = -2i$ |
| iii. $x = -2$ | vi. $x = 2i$ |

Which of these are zeros of the function $f(x) = x^4 - 16$?

- (A) iii and iv only (B) i, ii, iii, and iv only (C) v and vi only (D) i, ii, v, and vi only
 (E) None of the answers (A)–(D) is correct.

30. Given the function $f(x) = 12x - 7$, find a formula for $f^{-1}(x)$.

- (A) $x + \frac{7}{12}$ (B) $\frac{1}{12}(x + 7)$ (C) $\frac{x-7}{12}$ (D) $12x + 7$
 (E) None of the answers (A)–(D) is correct.