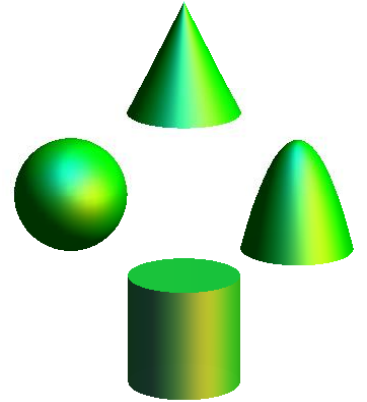# 2023
High School Math Contest

Level 3 Exam

Lenoir-Rhyne University

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This exam was prepared by
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Do's and Don'ts:

- **Do NOT** open this booklet until you are instructed to do so.

- **NO Calculators** (**Or other electronic devices**)

- Contestants with electronic devices (on or off) will be **disqualified!**
  And their **team** will be **disqualified!**
  (The other team members will continue to participate in the individual contest.)

DIRECTIONS:

- **The Score Card:**
  Write:
  - Your **NAME** on the "**name line**" (of course).
  - **Level 3** on the "**subject line**".
  - Your **SCHOOL** on the "**Date Line**"

- **Clearly mark** ONE bubble using **#2 PENCIL**.
  - Light marks will be counted as unmarked!
  - **Completely erase** any changes.

- You **can** write on this test booklet. (But the test booklet will not be graded.)

- **Tie Breakers**: In case of ties, the person with the least number of *wrong answers* wins.
  (A *blank* is better than *incorrect!*)

- **The Exam: 30** problems, **70** minutes.

- Problems are NOT in order of increasing difficulty, so feel free to skip around. (Just be careful to mark your answers correctly on your score card!)

- **WAIT** for the signal to begin.

1. Given that $a^{1/n} = 3$ where a is a real number and n is an integer greater than 1, all of the following are possible values of a **except** which one?

   (A) 3   (B) 9   (C) 27   (D) 81   (E) None of the answers (A)–(D) is correct.

2. How many zeros (including complex and counting multiplicity) does the function $h(x) = 7x^2 - 3x^3 + 7 - 12x$ have?

   (A) 2   (B) 3   (C) 6   (D) 7   (E) None of the answers (A)–(D) is correct.

3. Consider the function
$$f(x) = \begin{cases} 3x^2 + x - 4 & \text{for } x < -3, \\ -2x + 4 & \text{for } x \geq -3. \end{cases}$$

   Find $f(-2)$.

   (A) 0   (B) 6   (C) 8   (D) 10   (E) None of the answers (A)–(D) is correct.

4. Let x be the remainder when $n^2 + 10n + 18$ is divided by $n + 5$, and let y be the remainder when $m^2 - 28$ is divided by $m - 5$. What is value of xy?

   (A) 21   (B) -21 (C) -279 (D) 279   (E) None of the answers (A)–(D) is correct.

5. Below is a list of possible zeros for a function.

   i. $x = 16$
   ii. $x = -16$
   iii. $x = 4$

   iv. $x = -4$
   v. $x = \frac{3}{2}$
   vi. $x = -\frac{3}{2}$

   Which of the zeros in the list above are zeros of the function $f(x) = (2x - 3)(x + 4)^2$.

   (A) i and v only   (B) iii and vi only   (C) iv and v only   (D) iv and vi only

   (E) None of the answers (A)–(D) is correct.

6. Riley is 9 years old and her mother is 35 years old. How many years until Riley's mother is 3 times her age?

   (A) 4   (B) 5   (C) 7   (D) 13   (E) None of the answers (A)–(D) is correct.

7. If x is any number other than 4 or -8, what is the most simplified version of the following expression?
$$\frac{x^2 + 12x + 32}{x + 8} \div \frac{x^2 - 16}{2x - 8}.$$

   (A) 2   (B) $\frac{1}{2}$ (C) $\frac{2x-8}{x+8}$ (D) $\frac{(x+4)^2}{2}$ (E) None of the answers (A)–(D) is correct.

8. The period, T, of a simple pendulum is the time it take to complete a full swing back-and-forth. The factors that influence the period are L, the length of the pendulum, and g, the acceleration due to gravity, and these quantities are related by the equation
$$T = 2\pi\sqrt{\frac{L}{g}}.$$

   If the acceleration due to gravity is 9.8 meters per seconds squared and the period of a simple pendulum is 3 seconds, what is the length of the pendulum?

   (A) $2\pi\sqrt{\frac{3}{9.8}}$ meters   (B) $9.8\left(\frac{3}{2\pi}\right)^2$ meters   (C) $(3 - 2\pi)^2(9.8)$ meters

   (D) $\frac{3}{2\pi}\sqrt{9.8}$ meters   (E) None of the answers (A)–(D) is correct.

9. Simplify the expression $\left(\dfrac{3}{x+4}\right) - \left(\dfrac{5x-7}{x^2-16}\right)$.

   (A) $\frac{-2x-5}{x^2-16}$ (B) $\frac{-2x-19}{x^2-16}$ (C) $\frac{8x-19}{x^2-16}$ (D) $\frac{-5x+4}{x^2-16}$

   (E) None of the answers (A)–(D) is correct.

10. Solve the equation $125 = 25^{x+5}$.

   (A) $\frac{7}{2}$ (B) $-\frac{7}{2}$ (C) -3 (D) 0   (E) None of the answers (A)–(D) is correct.

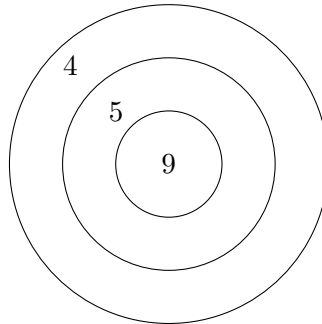11. Suppose x and y are solutions to the following equations.

$$3^{x-y} = 9$$
$$3^{x+y} = 27$$

What is the value of $x + 3y$?

(A) 3   (B) 4   (C) 9   (D) 15   (E) None of the answers (A)–(D) is correct.

12. Liam had six shots at the target shown below.

4

5

9

(Assume if he hits a line, it counts as the higher score.) Which of the following scores could Liam not have gotten?

(A) 43   (B) 44   (C) 45   (D) 46   (E) None of the answers (A)–(D) is correct.

13. A pilot flies her plane with the wind for 2 hours to make a 600 mile trip. Her return flight is against the wind and required 3 hours. Assuming the average wind speed and the average plane air speed each remained constant on both flights, what was the average wind speed?

(A) 25 mph   (B) 50 mph   (C) 75 mph   (D) 100 mph

(E) None of the answers (A)–(D) is correct.

14. What is the amplitude of the function $f(x) = 4\sin(2x) + 3$?

(A) 2   (B) π (C) 4   (D) 8   (E) None of the answers (A)–(D) is correct.

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15. Below is the graph of a function of the form $f(x) = a \cdot b^x$.
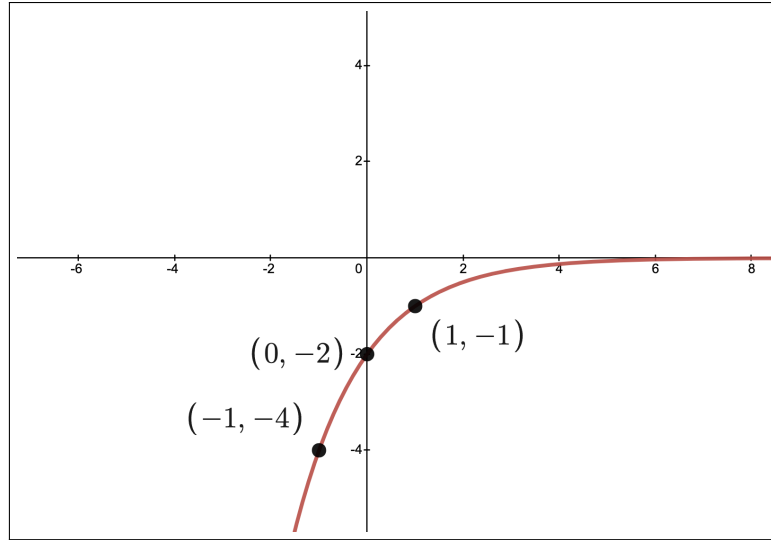
What is the value of $b - a$?

(A) -2.5 (B) -1 (C) 0 (D) 2.5 (E) None of the answers (A)–(D) is correct.

16. Below is the graph of a polynomial, including all of its zeros.
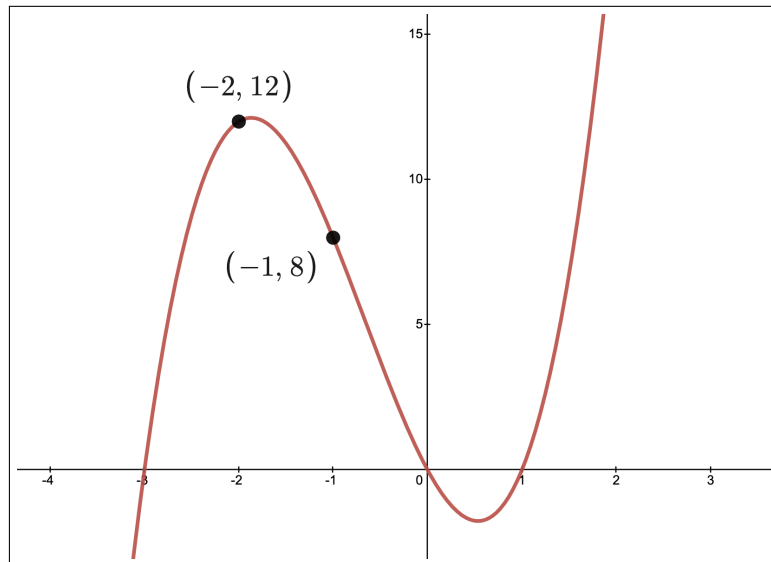
Assuming no multiple roots, what is the leading coefficient of the polynomial?

(A) 1 (B) 2 (C) 3 (D) 4 (E) None of the answers (A)–(D) is correct.

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17. Tyler bought a brand new 2019 Ford F-150 in 2019 for \$50,000. The value of the truck depreciated by a consistent percent each year. In two years, the truck was worth \$40,500. What is the annual rate of depreciation for his truck?

(A) 5%   (B) 7%   (C) 10%   (D) 15%   (E) None of the answers (A)–(D) is correct.

18. Which of the following logarithms have the same value?

  i. $\log_3(9)$ iii. $\log_5(25)$

  ii. $\log_4(10)$ iv. $\log_7(49)$

(A) i and ii only   (B) iii and iv only   (C) ii and iii only   (D) i, iii, and iv only

(E) None of the answers (A)–(D) is correct.

19. The point P is the incenter of $\triangle ABC$ below, and the dashed line segments from P are perpendicular to the triangle's sides.

If the length of $\overline{DC}$ is 12 units and the length of $\overline{PC}$ is 13 units, what is the length of $\overline{EP}$?

(A) 6   (B) 7   (C) 8   (D) 9   (E) None of the answers (A)–(D) is correct.

20. The parallelogram $QRST$ is given below with its diagonals intersecting at point P.
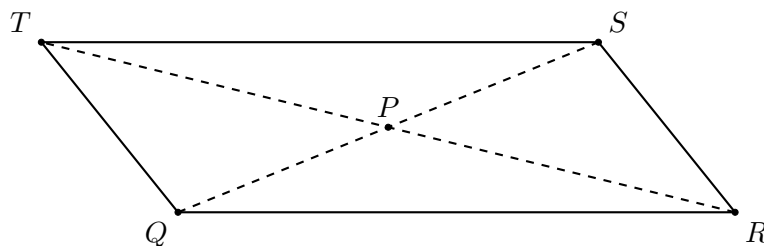
If the length of $\overline{RP}$ is $3(x+8)$, the length of $\overline{QP}$ is $2(x+7)$, and the length of $\overline{SP}$ is $4(x-3)$, find the value of x.

(A) -10 (B) 1   (C) 13   (D) 36   (E) None of the answers (A)–(D) is correct.

21. Consider the sequence
$$16, 18, 9, 12, 4, 8, 2, \ldots.$$
Choose the best answer for what number comes next, following the pattern given.

(A) 4   (B) 5   (C) 6   (D) 7   (E) There is no pattern.

22. Suppose a sector of a circle with central angle 4.5 radians has arc length 18 mm. What is the diameter of the circle?

(A) $\frac{1}{4}$ mm   (B) 4 mm   (C) 8 mm   (D) 18 mm

(E) None of the answers (A)–(D) is correct.

23. What is the equation of a circle with center $(3, -4)$ and diameter 6?

(A) $(x+3)^2 + (y-4)^2 = 36$ (B) $(x-3)^2 + (y+4)^2 = 36$ (C) $(x+3)^2 + (y-4)^2 = 9$

(D) $(x-3)^2 + (y+4)^2 = 9$ (E) None of the answers (A)–(D) is correct.

24. The Town of Neighborly Way has a cylindrical water tower with a radius of 16 meters and a height of 8 meters. The Town of Friendly Folks has a cylindrical water tower with a diameter of 8 meters and a height of 16 meters. How many times larger is the volume of Neighborly Way's water tower than that of Friendly Folks'?

(A) 2 times larger   (B) 4 times larger   (C) 8 times larger   (D) 16 times larger

(E) None of the answers (A)–(D) is correct.

25. Suppose that if the measure of one side of a square is increased by 2 centimeters and the measure of the adjacent side is decreased by 2 centimeters, the area of the resulting rectangle is 21 square centimeters. Find the measure of one side of the original square.

(A) 5   (B) 6   (C) 6.5   (D) 3   (E) None of the answers (A)–(D) is correct.

26. A car dealership is advertising that you can get 50% off any car on the lot if you can throw a dart and land it in a grey area of the square dart board shown below (with the side lengths of the squares labeled). You do not have much experience throwing darts so you make a truly random throw. Given that the dart lands on the board, and not on a line, what is the chance that you get the discount?
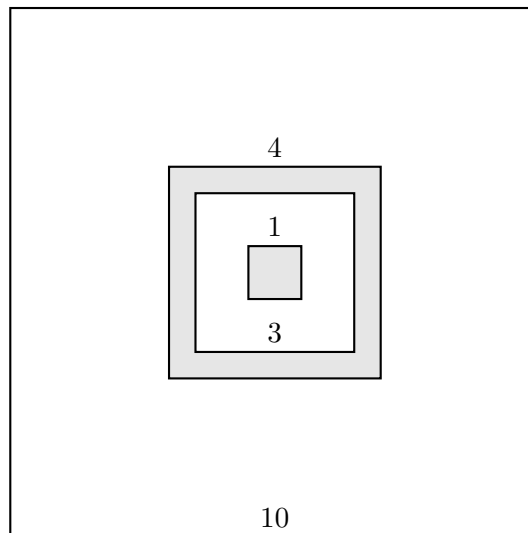
(A) 4%   (B) 7%   (C) 8%   (D) 16%   (E) None of the answers (A)–(D) is correct.

27. A new dance has come to any room with a tile floor. The dancer starts on a tile. The first move is going forward two tiles. The second move is going back one tile. The dancer repeats these two moves over and over — forward two, back one, forward two, back one, etc. How many moves will it take for the dancer to move forward 10 tiles from the starting tile?

(A) 17 moves   (B) 18 moves   (C) 19 moves   (D) 20 moves

(E) None of the answers (A)–(D) is correct.

28. Consider the following expressions.

   i. $\left(a^{\sqrt{3}}\right)^2$

   ii. $\left(\frac{a^5 \cdot a^7}{a^4}\right)^0 - \frac{(a^4)^4}{a^{16}} + a^3$

   iii. $a^3 \left(1 - a^{\frac{1}{3}}\right) + a^3 \left(a^{\frac{1}{6}}\right)^2$

   Which of these expressions are equivalent?

   (A) i and ii only   (B) i and iii only   (C) ii and iii only   (D) i, ii, and iii

   (E) none are equivalent

29. Below is a list of possible zeros for a function.

   i. $x = -4$ iv. $x = 2$

   ii. $x = 4$ v. $x = -2i$

   iii. $x = -2$ vi. $x = 2i$

   Which of these are zeros of the function $f(x) = x^4 - 16$?

   (A) iii and iv only   (B) i, ii, iii, and iv only   (C) v and vi only   (D) i, ii, v, and vi only

   (E) None of the answers (A)–(D) is correct.

30. Given the function $f(x) = 12x - 7$, find a formula for $f^{-1}(x)$.

   (A) $x + \frac{7}{12}$ (B) $\frac{1}{12}(x + 7)$ (C) $\frac{x-7}{12}$ (D) $12x + 7$

   (E) None of the answers (A)–(D) is correct.