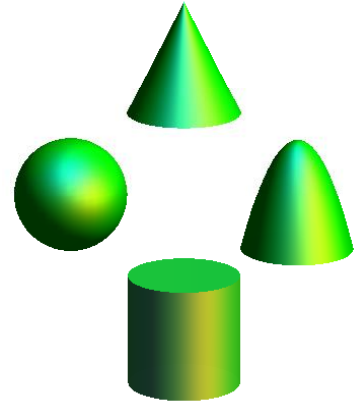
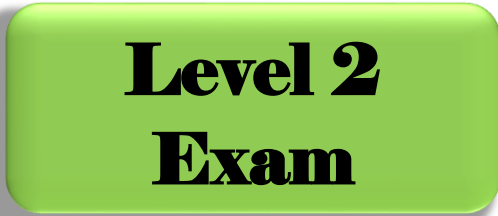




2023

High School Math Contest



Lenoir-Rhyne University

*Donald and Helen Schort School of
Mathematics and Computing Sciences*

This exam was prepared by
Stephanie Hays, CVCC Professor & LR Graduate
Assisted by Dr. Timothy Goldberg, LR Professor.

Do's and Don'ts:

- **Do NOT** open this booklet until you are instructed to do so.
- **NO Calculators** (Or other electronic devices)
- Contestants with electronic devices (on or off) will be **disqualified!**
And their **team** will be **disqualified!**
(The other team members will continue to participate in the individual contest.)

DIRECTIONS:

- **The Score Card:**
Write:
 - Your **NAME** on the “name line” (of course).
 - **Level 2** on the “subject line”.
 - Your **SCHOOL** on the “Date Line”
- **Clearly mark** ONE bubble using **#2 PENCIL**.
 - Light marks will be counted as unmarked!
 - **Completely erase** any changes.
- You **can** write on this test booklet. (But the test booklet will not be graded.)
- **Tie Breakers:** In case of ties, the person with the least number of *wrong answers* wins.
(A *blank* is better than *incorrect!*)
- **The Exam:** **30** problems, **70** minutes.
- Problems are NOT in order of increasing difficulty, so feel free to skip around. (Just be careful to mark your answers correctly on your score card!)

• **WAIT** for the signal to begin.

1. Suppose that if the measure of one side of a square is increased by 2 centimeters and the measure of the adjacent side is decreased by 2 centimeters, the area of the resulting rectangle is 21 square centimeters. Find the measure of one side of the original square.

(A) 5 (B) 6 (C) 6.5 (D) 3 (E) None of the answers (A)–(D) is correct.

2. The period, T , of a simple pendulum is the time it take to complete a full swing back-and-forth. The factors that influence the period are L , the length of the pendulum, and g , the acceleration due to gravity, and these quantities are related by the equation

$$T = 2\pi\sqrt{\frac{L}{g}}.$$

If the acceleration due to gravity is 9.8 meters per seconds squared and the period of a simple pendulum is 3 seconds, what is the length of the pendulum?

- (A) $2\pi\sqrt{\frac{3}{9.8}}$ meters (B) $9.8\left(\frac{3}{2\pi}\right)^2$ meters (C) $(3 - 2\pi)^2(9.8)$ meters (D) $\frac{3}{2\pi}\sqrt{9.8}$ meters
(E) None of the answers (A)–(D) is correct.

3. Consider $\triangle CAT$ with points $C(3, 4)$, $A(-2, -4)$, and $T(0, 5)$. What is the location of the image T' of the point T after the triangle is reflected over the line $y = 4$?

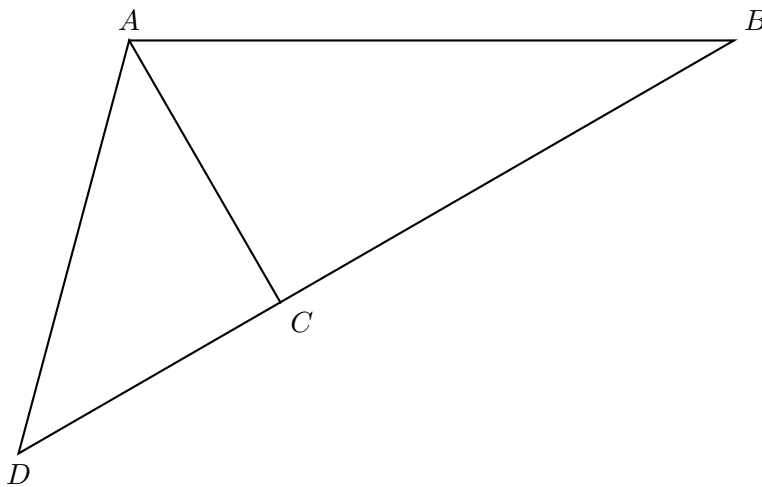
- (A) (8, 5) (B) (3, 4) (C) (0, -5) (D) (0, 3)
(E) None of the answers (A)–(D) is correct.

4. Consider again $\triangle CAT$ from the previous question. Which of the following transformations will create a triangle similar but not congruent to $\triangle CAT$?

- (A) Translation (B) Dilation (C) Reflection (D) Rotation
(E) None of the answers (A)–(D) is correct.

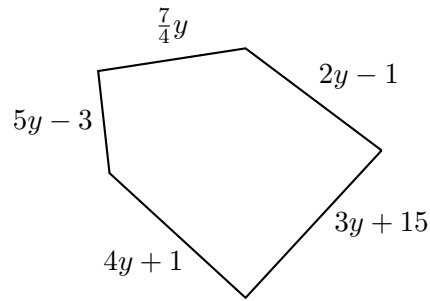
5. Riley lives in a city where the bus runs every 40 minutes. If she arrives at the bus stop at a random time, what is the probability that she will not have to wait more than 15 minutes?
- (A) $\frac{1}{4}$ (B) $\frac{3}{10}$ (C) $\frac{3}{8}$ (D) $\frac{5}{8}$
- (E) None of the answers (A)–(D) is correct.
6. A triangular sign on a billboard covers an area of 100 square feet. The sign has a base that is twice as long as its height. Find the base of the sign.
- (A) 10 feet (B) 20 feet (C) $33\frac{1}{3}$ feet (D) $66\frac{2}{3}$ feet
- (E) None of the answers (A)–(D) is correct.
7. A local car lot advertised that you could win a car by drawing two aces from a standard deck of cards. The first card is not replaced. What is the probability of winning a car? (Tip: A standard deck of cards consists of 52 cards. There are 4 aces in the whole deck.)
- (A) $\frac{4^2}{52^2}$ (B) $\frac{4 \cdot 4}{52 \cdot 51}$ (C) $\frac{4 \cdot 3}{52 \cdot 52}$ (D) $\frac{4 \cdot 3}{52 \cdot 51}$
- (E) None of the answers (A)–(D) is correct.
8. A box contains cards numbered from 1 to 10. What is the probability a card chosen at random is an odd number or a prime number?
- (A) 50% (B) 60% (C) 70% (D) 80% (E) None of the answers (A)–(D) is correct.

9. In the diagram below, \overline{AC} is an altitude of $\triangle ABD$, the length of \overline{AB} is 20, the measure of $\angle DAB$ is 105° , and the measure of $\angle ABC$ is 30° . Find the length of \overline{AD} . Write your answer in simplest radical form.



- (A) 20 (B) $10\sqrt{3}$ (C) $10\sqrt{2}$ (D) 10 (E) None of the answers (A)–(D) is correct.
10. If length and width of a rectangle are integers and the area of the rectangle is 64 square units, how many different lengths are possible?
- (A) 4 (B) 7 (C) 8 (D) Infinitely many (E) None of the answers (A)–(D) is correct.
11. Suppose n is a three-digit number that does not repeat any digits. The second digit is four times the third digit, while the first digit is three less than the second digit. What is the third digit of n ?
- (A) 1 (B) 2 (C) 3 (D) 4 (E) None of the answers (A)–(D) is correct.

12. Find an expression for the perimeter of the figure below.



- (A) $21y + 12$ (B) $\frac{63}{4}y + 12$ (C) $21y + 20$ (D) $\frac{63}{4}y + 20$
(E) None of the answers (A)–(D) is correct.
13. If u varies inversely as square root of w , and $u = 8$ when $w = 49$, find u when $w = 121$.
- (A) $\frac{392}{121}$ (B) $\frac{11}{56}$ (C) $\frac{56}{11}$ (D) $\frac{8}{27}$ (E) None of the answers (A)–(D) is correct.
14. The diagonal measurement of a laptop screen is 15 inches, and its length is 3 inches more than its width. What is the length of the laptop?
- (A) 5 inches (B) 6 inches (C) 9 inches (D) 12 inches (E) None of the answers (A)–(D) is correct.
15. In a triangle, the second angle is 20% more than the first angle, and the third angle is 20% less than the first angle. Find the smallest angle of the triangle.
- (A) 40° (B) 48° (C) 60° (D) 72° (E) None of the answers (A)–(D) is correct.

16. Consider the following expressions.

- i. $a^{\sqrt{2}} \cdot a^{\sqrt{2}}$
- ii. $a^0 \left(\frac{a^4 \cdot a^3}{a^5} \right)$
- iii. $a^2 \left(1 - a^{\frac{1}{3}} \right) + a^2 \left(a^{\frac{1}{6}} \right)^2$

Which of these expressions are equivalent?

- (A) i and ii only (B) i and iii only (C) ii and iii only (D) i, ii, and iii (E) None are equivalent.

17. Observe the following steps of solving the quadratic by completing the square.

- Problem: $x^2 + 6x - 7 = 0$
Step 1: $x^2 + 6x = 7$
Step 2: $x^2 + 6x + 9 = 16$
Step 3: $(x + 3)^2 = 16$
Step 4: $x + 3 = \pm 4$
Step 5: $x = -7, 1$

Which step contains an error?

- (A) Step 2 (B) Step 3 (C) Step 4 (D) Step 5 (E) There is no error.

18. Consider the expression

$$-i^2 + 3i - 3(1 - i).$$

What is the sum of the real and complex components of the simplified expression?

- (A) 4 (B) -4 (C) -2 (D) 2 (E) None of the answers (A)–(D) is correct.

19. A 25-foot ladder leans against a building so that the angle between the ground and the ladder is 60° . How high does the ladder reach up the side of the building?

- (A) $25\sqrt{3}$ feet (B) $\frac{25\sqrt{2}}{2}$ feet (C) $\frac{25}{2}$ feet (D) $\frac{25\sqrt{3}}{2}$ feet (E) None of the answers (A)–(D) is correct.

20. The squares of two positive integers add to 117. The second integer is twice the value of the square root of the first integer. What is the sum of the two integers?

- (A) 8 (B) 9 (C) 12 (D) 15 (E) None of the answers (A)–(D) is correct.

21. Among all of the pairs of real numbers whose sum is 7, one pair has the largest product. What is this largest product?

- (A) 6 (B) 7 (C) 10 (D) 12 (E) None of the answers (A)–(D) is correct.

22. The graph of

$$y = f(x + 5) - 3$$

is a transformation of which two transformations of the graph of the original function $y = f(x)$?

- i. Vertical shift down 3 unit
- ii. Horizontal shift right 5 units
- iii. Horizontal shift left 5 units
- iv. Horizontal compression by a factor of $1/5$
- v. Vertical stretch by a factor of 5

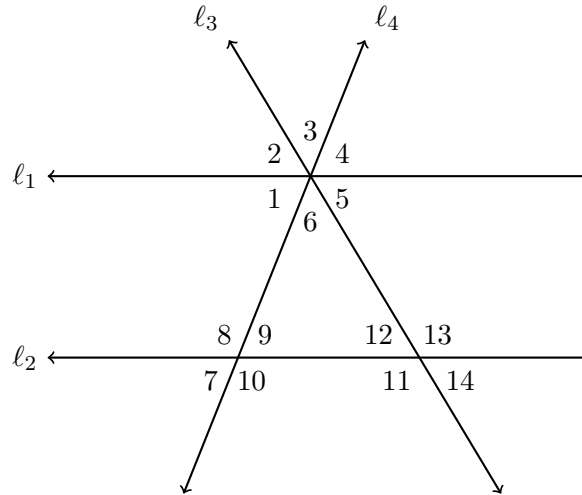
- (A) i and ii only (B) i and iii only (C) i and iv only (D) i and v only (E) None of the answers (A)–(D) is correct.

23. Given that $a^{1/n} = 3$ where a is a real number and n is an integer greater than 1, all of the following are possible values of a **except** which one?

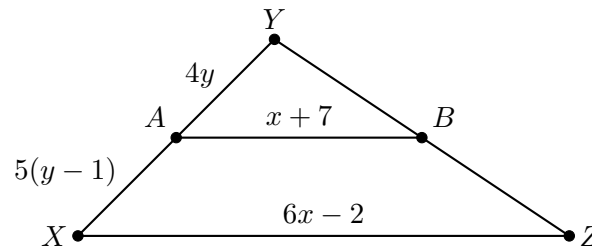
- (A) 3 (B) 9 (C) 27 (D) 81 (E) None of the answers (A)–(D) is correct.

24. Suppose ℓ_1 and ℓ_2 are parallel lines, and ℓ_3 and ℓ_4 are any two lines not parallel to ℓ_1 and ℓ_2 , so that ℓ_1 , ℓ_3 , and ℓ_4 are concurrent as in the figure below.

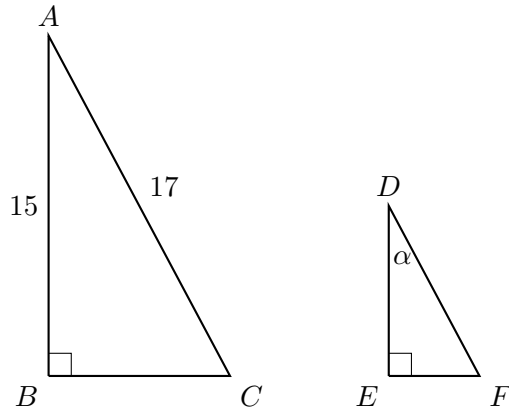
Which of angles 2–14 below must be congruent to angle 1?



- (A) 4 only (B) 4, 7, and 9 only (C) 2, 4, 5, 7, 9 only (D) 2, 4, 5, 7, 9, 12, and 14 only
 (E) None of the answers (A)–(D) is correct.
25. In the figure below, point A is the midpoint of side \overline{XY} and B is the midpoint of side \overline{YZ} . What is sum of x and y ?



- (A) 10 (B) 4 (C) 9 (D) -9 (E) None of the answers (A)–(D) is correct.
26. In the following diagram, $\triangle ABC$ is similar to $\triangle DEF$.



What is the measure of angle α ?

- (A) $\cos^{-1}\left(\frac{8}{17}\right)$ (B) $\tan^{-1}\left(\frac{15}{17}\right)$ (C) $\cos^{-1}\left(\frac{15}{17}\right)$ (D) $\sin^{-1}\left(\frac{15}{17}\right)$ (E) None of the answers (A)–(D) is correct.

27. A total of 100 people were asked, “Do you prefer cats or dogs?” The results are shown below, along with the handedness of the people.

	Cats	Dogs	Total
Left-Handed	25	20	45
Right-Handed	15	40	55
Total	40	60	100

Based on this data, what is the probability that a person surveyed preferred cats, given that the individual was left-handed?

- (A) $\frac{1}{4}$ (B) $\frac{2}{5}$ (C) $\frac{5}{9}$ (D) $\frac{5}{8}$ (E) None of the answers (A)–(D) is correct.

28. Find all possible solutions to the equation

$$x^2 - x = -36$$

- (A) ± 6 (B) $\pm 6i$ (C) $\frac{1 \pm i\sqrt{143}}{2}$ (D) $\frac{1}{2} \pm i\frac{\sqrt{145}}{2}$ (E) There are no solutions.

29. Let x be the remainder when $n^2 + 10n + 18$ is divided by $n + 5$, and let y be the remainder when $m^2 - 28$ is divided by $m - 5$. What is value of xy ?

(A) 21 (B) -21 (C) -279 (D) 279 (E) None of the answers (A)–(D) is correct.

30. Solve the equation $125 = 25^{x+5}$.

(A) $\frac{7}{2}$ (B) $-\frac{7}{2}$ (C) -3 (D) 0 (E) None of the answers (A)–(D) is correct.

