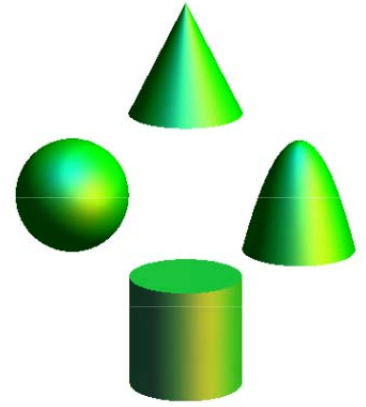




**2015**  
**High School Math Contest**

**Level 3**  
**Exam**



**Lenoir-Rhyne University**

*Donald and Helen Schort School of  
Mathematics and Computing Sciences*

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**Do's and Don'ts:**

- **Do NOT** open this booklet until you are instructed to do so.
- **NO Calculators (Or other electronic devices)**
- Contestants with electronic devices (on or off) will be **disqualified!**  
And their **team** will be **disqualified!**  
(The other team members will continue to participate in the individual contest.)

**DIRECTIONS:**

• **The Score Card:**

Write:

- Your **NAME** on the “name line” (of course).
- “**Level 3**” on the “subject line”.
- Your **SCHOOL** on the “**Date Line**”
- **Clearly mark** ONE bubble using **#2 PENCIL**.
  - Light marks will be counted as unmarked!
  - **Completely erase** any changes.
- You **can** write on this test booklet. (But the test booklet will not be graded.)
- **Tie Breakers:** In case of ties, the person with the least number of *wrong answers* wins.  
(A *blank* is better than *incorrect!*)
- **The Exam:** **30** problems, **70** minutes.

- **WAIT** for the signal to begin.

1. A quadratic equation  $y = ax^2 + bx + c$  is known to pass through the points  $(-1, 5)$ ,  $(0, -1)$ , and  $(3, 5)$ . Find the sum of the coefficients.

(A)  $-1$  (B)  $2$  (C)  $9$  (D)  $-3$  (E) None of the answers (A)–(D) is correct.

2. If  $x^2 + 2Ax + B = (x - A)(x - B)$  for all numbers  $x$ , and  $A \neq B$ , then  $B$  is equal to

(A)  $-3$  (B)  $0$  (C)  $1$  (D)  $3$  (E) None of the answers (A)–(D) is correct.

3. Simplify the expression  $\sqrt{\frac{\sqrt{5}-2}{\sqrt{5}+2}}$ .

(A)  $\frac{1}{\sqrt{5}-\sqrt{2}}$  (B)  $\frac{1}{\sqrt{5}-2}$  (C)  $\sqrt{5}-2$  (D)  $\sqrt{5}+2$

(E) None of the answers (A)–(D) is correct.

4. If  $2x + 5y = 3$  and  $4x^2 - 25y^2 = 24$ , find the value of  $x - \frac{5}{2}y$ .

(A)  $2$  (B)  $4$  (C)  $8$  (D)  $16$  (E) None of the answers (A)–(D) is correct.

5. If  $(\sqrt[3]{2})^{2-x} = 2^{x^2}$ , then  $x$  has two solutions  $a$  and  $b$ . What is  $a + b$ ?

(A)  $-\frac{2}{3}$  (B)  $-\frac{1}{3}$  (C)  $\frac{1}{3}$  (D)  $\frac{2}{3}$  (E) None of the answers (A)–(D) is correct.

6. Find the equation of the line perpendicular to the line  $2x + 4y = -3$  that passes through the center of the circle  $x^2 + 6x + y^2 - 8y = 36$ .

(A)  $y = -2x - 2$  (B)  $y = -2x + 2$  (C)  $y = 2x - 10$  (D)  $y = 2x + 10$

(E) None of the answers (A)–(D) is correct.

7. Find the general expression of a quadratic function whose  $x$ -intercepts are  $(-4, 0)$  and  $(2, 0)$  and whose range is  $y \geq -18$ .
- (A)  $f(x) = x^2 + 2x - 8$  (B)  $f(x) = x^2 - 2x + 8$  (C)  $f(x) = 2x^2 + 4x - 16$  (D)  $f(x) = 2x^2 - 4x + 16$   
 (E) None of the answers (A)–(D) is correct.
8. Mary has three marbles of different colors, but the same size, in a bag. She randomly chooses a marble from the bag, looks at it, and puts it back. She repeats the process two more times. What is the probability that she has seen at most two different marbles?
- (A)  $7/9$  (B)  $1/3$  (C)  $2/9$  (D)  $15/16$  (E) None of the answers (A)–(D) is correct.
9. If  $f(x) = 2^x$  then  $4^8$  is equal to:
- (A)  $f(f(2))$  (B)  $f(f(f(2)))$  (C)  $f(f(f(f(2))))$  (D)  $f(f(f(f(f(2)))))$   
 (E) None of the answers (A)–(D) is correct.
10. If  $\frac{(2+i)(3+2i) - (1+2i)(x+yi)}{2+i} = 1$ , calculate  $\frac{x}{y}$ .
- (A) 9 (B) 14 (C) 6 (D) 7 (E) None of the answers (A)–(D) is correct.
11. The equation  $8^{x^2+3x} = 32^{-3x+3}$  has two solutions  $a$  and  $b$ . What is  $a \cdot b$ ?
- (A)  $-3$  (B)  $-4$  (C)  $-5$  (D)  $-6$  (E) None of the answers (A)–(D) is correct.
12. Solve for  $x$ :  $\log_8(x+3) = \log_{16}(81)$ .
- (A) 24 (B)  $75/2$  (C) 6 (D) 78 (E) None of the answers (A)–(D) is correct.

13. Suppose a function  $f(x)$  satisfies  $f(x) + 4f(3 - x) = 5x$  for all real numbers  $x$ . Then  $f(3)$  is  
(A)  $-3$  (B)  $-2$  (C)  $0$  (D)  $4$  (E) None of the answers (A)–(D) is correct.
14. Which of the following are true about the transformation of  $f(x)$  to  $g(x)$  if  $f(x) = x^2$  and  $g(x) = 3x^2 - x + 4$ ?  
(A) Shifted up by  $\frac{1}{12}$ . (B) Shifted right by  $\frac{1}{6}$ . (C) Shifted left by  $\frac{1}{6}$ .  
(D) Both (A) and (B) are correct. (E) Both (A) and (C) are correct.
15. One of Jason and Jerry lies on Mondays, Tuesdays and Wednesdays, and tells the truth on the other days of the week. The other lies on Thursdays, Fridays and Saturdays, and tells the truth the other days of the week. At noon, the two had the following conversation:  
**Jerry:** I lie on Saturdays.  
**Jason:** I will lie tomorrow.  
**Jerry:** I lie on Sundays.  
This conversation takes place on a:  
(A) Monday (B) Tuesday (C) Wednesday (D) Friday (E) Saturday
16. A square is inscribed within a circle. If the ratio of circumference to area of the circle is 7, what is the ratio of perimeter to area for the square?  
(A)  $7\sqrt{2}$  (B)  $2\sqrt{7}$  (C)  $\frac{7}{\sqrt{2}}$  (D)  $\frac{2}{\sqrt{7}}$  (E) None of the answers (A)–(D) is correct.
17. I want to triple the volume in a cylindrical can. Due to package limitations, I can only increase the height of the can by 50%. Assuming I do this, approximate the percentage that the radius of the can must be increased to triple the volume.  
(A) 41.4% (B) 45.2% (C) 52.1% (D) 53.8% (E) None of the answers (A)–(D) is correct.

18. Let  $O$  be the center of a circular pond of radius 7 meters. The point  $A$  is on the edge of the pond, due east of  $O$ . A boy swims westward from  $A$  to the point  $B$ , 3 meters from  $A$ . Then he turns north and swims to shore, whereupon he swims westward again until he reaches the point  $C$  due north of  $O$ . The distance in meters between  $O$  and  $C$  is:
- (A)  $2\sqrt{7}$  (B) 4 (C)  $7\sqrt{3}$  (D)  $\sqrt{33}$  (E) None of the answers (A)–(D) is correct.
19. Which of the following angles does not share its terminal side with the others?
- (A)  $-\frac{310\pi}{45}$  (B)  $1640^\circ$  (C)  $-3760^\circ$  (D)  $-\frac{62\pi}{9}$
- (E) None of the answers (A)–(D) is correct.
20. You drive a car with 20-inch diameter tires and are traveling on the turnpike at a constant speed of 80 mph. If you have been traveling under these conditions for 30 minutes, how many complete revolutions did your tires make?
- (A)  $\frac{173200}{\pi}$  (B)  $\frac{153190}{\pi}$  (C)  $\frac{146430}{\pi}$  (D)  $\frac{132380}{\pi}$
- (E) None of the answers (A)–(D) is correct.
21. A survey of students taking both college algebra and statistics at a university found that 70% received a passing grade in college algebra, 75% received a passing grade in statistics, and 60% passed both courses. If a person received a passing grade in statistics, what is the probability that the person also passed college algebra?
- (A) 60% (B) 70% (C) 80% (D) 90% (E) None of the answers (A)–(D) is correct.
22. Find the solution of the inequality  $2 - |x - |5 - 4x| + 3| \leq 0$ .
- (A)  $(-\infty, 0] \cup [\frac{2}{3}, 4] \cup [\frac{15}{2}, \infty)$  (B)  $(-\infty, 0] \cup [\frac{4}{5}, 2] \cup [\frac{10}{3}, \infty)$  (C)  $(-\infty, 0] \cup [\frac{3}{2}, 3] \cup [\frac{11}{3}, \infty)$
- (D)  $(-\infty, 0] \cup [\frac{5}{4}, 4] \cup [\frac{16}{3}, \infty)$  (E) None of the answers (A)–(D) is correct.

23. Let  $N$  be the sum of all prime numbers which divide 2015. Then the number of distinct prime numbers which divide  $N$  is:

- (A) 1 (B) 2 (C) 3 (D) 4 (E) None of the answers (A)–(D) is correct.

24. Simplify the following:

$$(\log_{128} 1331)(\log_{729} 1024)(\log_{3125} 81)(\log_{121}(5^{-1}))$$

- (A)  $-\frac{15}{28}$  (B)  $-\frac{2}{7}$  (C)  $-\frac{5}{8}$  (D)  $-\frac{5}{14}$  (E) None of the answers (A)–(D) is correct.

25. Town A and Town B are 225 miles apart. Car A leaves Town A at noon, and drives toward Town B at 60 mph. At 1pm, Car B leaves Town B at 45 mph, driving along the same road toward Town A. The speed limit for the road is set at 50mph. After driving 20 miles, Car A is pulled over by the police and cited for speeding. After an additional 22 minutes, he is given a ticket and can continue driving toward Town B, albeit driving a constant 50 mph. At what time do the cars meet?

- (A) 2:40 pm (B) 2:45 pm (C) 2:50 pm (D) 3:00 pm  
(E) None of the answers (A)–(D) is correct.

26. Recall that all consecutive elements of an arithmetic sequence satisfy  $a_{n+1} = a_n + d$  for some constant  $d$ . Suppose that the sum of the first and fourth terms of an increasing arithmetic sequence is 2, and the sum of their squares is 20. Find the sum of the first eight terms of the sequence.

- (A)  $\frac{178}{3}$  (B)  $\frac{184}{3}$  (C) 38 (D) 40 (E) None of the answers (A)–(D) is correct.

27. One root of  $f(x) = x^4 - 5x^3 + (6 - k)x^2 + 5kx - 6k$  is  $x = 2$ . There are two nonzero values of  $k$ , ( $k_1 < k_2$ ), for which the roots of  $f(x)$  are not unique. What is their ratio,  $\frac{k_2}{k_1}$ ?

- (A) 9/4 (B) 3/2 (C) 2 (D) 4/3 (E) None of the answers (A)–(D) is correct.

28. Suppose that  $f(x) = \frac{3x+1}{x-2}$  and  $g(x) = x^3 + x - 2$ . Note that both  $f(x)$  and  $g(x)$  are invertible. Calculate  $[(f^{-1}) \circ (g^{-1})](-2)$ .

- (A)  $\frac{4}{3}$    (B)  $\frac{3}{5}$    (C)  $-\frac{1}{3}$    (D)  $-\frac{3}{2}$    (E) None of the answers (A)–(D) is correct.

29. A twelve-hour digital watch displays the hours and minutes. For how long during one complete day does the watch display at least one 4?

- (A) 1 hour 15 minutes   (B) 5 hours and 24 minutes   (C) 6 hours and 10 minutes  
(D) 7 hours and 30 minutes   (E) None of the answers (A)–(D) is correct.

30. Let  $\lfloor x \rfloor$  denote the largest integer not exceeding  $x$ . Which of the following statements are always true?

$$\begin{aligned} I : & \quad \lfloor x - y \rfloor = \lfloor x \rfloor - \lfloor y \rfloor \\ II : & \quad \lfloor 3x \rfloor = 3\lfloor x \rfloor \\ III : & \quad \lfloor -x \rfloor = -\lfloor x \rfloor \end{aligned}$$

- (A) *I* only   (B) *II* only   (C) *III* only   (D) all of the statements are true  
(E) none of the statements are true