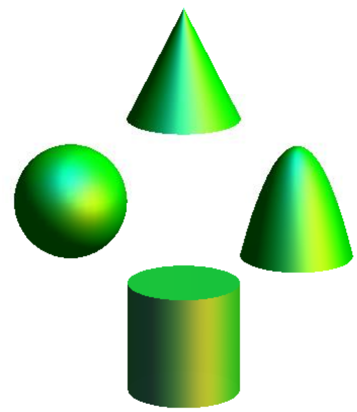
2017
High School Math Contest

Level 2 Exam Key



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1. Simplify the statement $\left[(5\times 2-12\div 2)^2-8\div 4\right]\div\left[-2+(3\times 4-14)^2\right]$

(A) $\frac{-1}{2}$ *(B) 7* (C) 12 (D) $\frac{-7}{8}$ (E) None of the answers (A) through (D) is correct.

Solution:

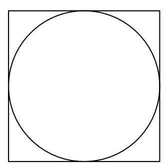
$$\begin{aligned}\left[(5\times 2-12\div 2)^2-8\div 4\right]\div\left[-2+(3\times 4-14)^2\right] &= \left[(4)^2-8\div 4\right]\div\left[-2+(-2)^2\right]\\ &= [16-8\div 4]\div[-2+4]\\ &= [16-2]\div[2]\\ &= 14\div 2\\ &= 7\end{aligned}$$

2. Which of the following is equivalent to the equation $y=3^{1/4}$?

(A) $\log_y 3=\frac{1}{4}$ *(B) $\log_3 y=\frac{1}{4}$* (C) $\log_{\frac{1}{4}} 3=y$ (D) $\log_{\frac{1}{4}} y=3$ (E) None of the answers (A) through (D) is correct.

Solution: The equation $y=3^{1/4}$ is equivalent to $\log_3 y=\frac{1}{4}$.

3. In the figure below, the square has area 36. Find the area of the inscribed circle.



(A) 36π (B) 6π (C) 3π *(D) 9π* (E) None of the answers (A) through (D) is correct.

Solution: Since the area of the square is 36, each side has length 6. The radius of the circle is half the side length of the square, so the radius is 3. Hence the area of the circle is $\pi(3^2)=9\pi$.

4. Find the inverse function of $y=\frac{2x+1}{5}$.

(A) $y=\frac{x-2}{5}$ (B) $y=\frac{\frac{1}{2}x-1}{\frac{1}{5}}$ *(C) $y=\frac{5x-1}{2}$* (D) $y=\frac{xy+1}{2}$ (E) None of the answers (A) through (D) is correct.

Solution: First we exchange x and y in the original function and obtain $x=\frac{2y+1}{5}$. Then we solve for y, yielding $y=\frac{5x-1}{2}$.

5. A right triangle has hypotenuse length 4 and another side length 2. Find the area of this triangle.

(A) $2\sqrt{3}$ (B) $4\sqrt{3}$ (C) $4\sqrt{2}$ (D) $2\sqrt{2}$ (E) None of the answers (A) through (D) is correct.

Solution: Let x be the length of the third side of the right triangle. By the Pythagorean theorem we have $x^2 + 2^2 = 4^2$, so $x^2 = 16 - 4 = 12$, so $x = \sqrt{12} = 2\sqrt{3}$. The area of the triangle is $\frac{1}{2}(2)(2\sqrt{3}) = 2\sqrt{3}$.

6. 100 Freshmen were asked to select either statistics or calculus as a math course to take next semester. These students were also asked to select chemistry or astronomy as a science course to take next semester. Below is a contingency table that represents the selections these students made:

	Statistics	Calculus
Chemistry	26	15
Astronomy	42	17

Using the table above, what is the probability that a student chose to take calculus or the student chose to take astronomy?

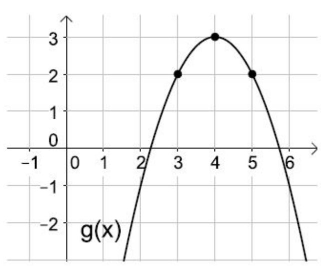
(A) $\frac{17}{20}$ (B) $\frac{29}{50}$ *(C)* $\frac{37}{50}$ (D) $\frac{17}{100}$ (E) None of the answers (A) through (D) is correct.

Solution:

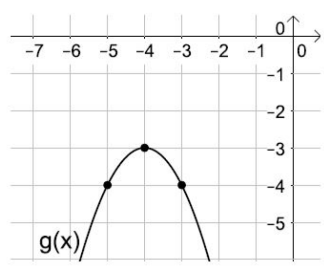
$$\begin{aligned} P(\text{calculus} \cup \text{astronomy}) &= \frac{59 + 32 - 17}{100} \\ &= \frac{37}{50} \end{aligned}$$

7. Suppose $f(x) = x^2$. Which graph represents $g(x) = -f(x-4) + 3$, a transformation of the function $f(x)$?

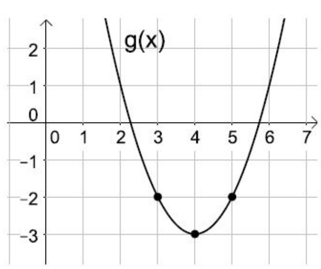
I



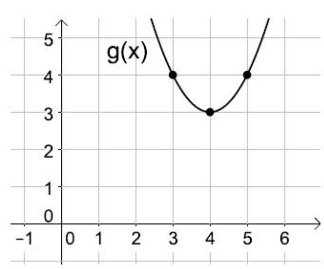
II



III



IV



(A) graph I (B) graph II (C) graph III (D) graph IV
(E) None of the answers (A) through (D) is correct.

Solution: The transformations on $f(x)$ that create $g(x)$ are as follows: shifted right 4, reflected over the x-axis, and shifted up 3. Of the given graphs, only two of them are shifted right 4 and up 3. Graph I opens down, which represents a reflection over the x-axis. Therefore, the solution is Graph I

8. Find the equation of the line passing through the point $(8, 2)$ and perpendicular to $4x + 3y = 9$.

   (A) $y = \frac{3}{4}x + 2$ (B) $y = \frac{-3}{4}x + 3$ (C) $y = \frac{4}{3}x + 5$ (D) $y = \frac{-4}{3}x + 1$
   (E) None of the answers (A) through (D) is correct.

Solution: The line can be written as $y = \frac{-4}{3}x + 3$. The slope of a perpendicular line is $m = \frac{3}{4}$. Using the point-slope form of a line, we get $y - 2 = \frac{3}{4}(x - 8)$ or $y = \frac{3}{4}x - 4$

9. Given the equation of a circle, $x^2 + 6x + y^2 - 8y = 11$, find the distance between the center of the circle and the origin.

   (A) 25 (B) $\sqrt{5}$ (C) 6 *(D) 5* (E) None of the answers (A) through (D) is correct.

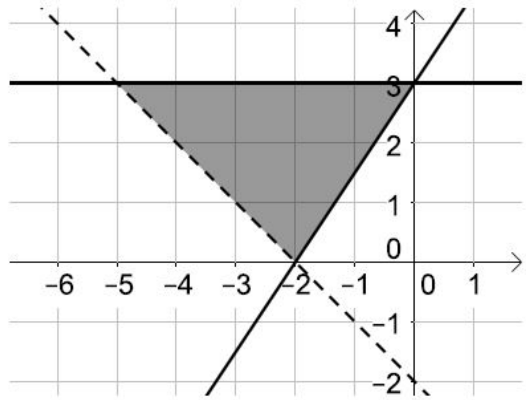
Solution: First find the center of the circle by completing the square:

$$\begin{aligned} x^2 + 6x + y^2 - 8y &= 11 \\ x^2 + 6x + 9 - 9 + y^2 - 8y + 16 - 16 &= 11 \\ (x + 3)^2 + (y - 4)^2 &= 36 \end{aligned}$$

Therefore the center of the circle is located at (-3,4). Now we can find the distance:

$$\begin{aligned} d &= \sqrt{(0 + 3)^2 + (0 - 4)^2} \\ &= \sqrt{25} \\ &= 5 \end{aligned}$$

10. Which inequalities, listed below the graph, are true about the system of equations that would represent the graph shown below?



I: $y \leq 3$
II: $y \geq \frac{3}{2}x + 3$
III: $y \geq -x - 2$

(A) Only I is true (B) Only III is true (C) Both II and III are true (D) I, II, and III are true
(E) None of the answers (A) through (D) is correct.

Solution: In the given figure, the shaded region is below the line $y = 5$, so inequality I is correct, since the line is to be included. The shaded region is above the line $y = \frac{3}{2}x + 3$, so inequality II is correct as the line is also to be included. The shaded region is above the line $y = -x - 2$, so inequality III is incorrect as the line is not to be included. Therefore, I and II are true while III is false, which leads to none of the above.

11. Which of the following tables represent two events, A and B, that are independent?

I

	A	∼ A
B	17	41
∼ B	16	26

II

	A	∼ A
B	15	45
∼ B	10	30

III

	A	∼ A
B	20	37
∼ B	33	10

(A) I only *(B) II only* (C) III only (D) Both I and III
(E) None of the answers (A) through (D) is correct.

Solution: Events are independent if $P(A|B) = P(A)$.

For Table I:

$$P(A|B) = \frac{17}{58}$$
$$P(A) = \frac{33}{100}$$
$$P(A|B) \neq P(A) \text{ dependent}$$

For Table II:

$$P(A|B) = \frac{15}{60}$$
$$P(A) = \frac{25}{100}$$
$$P(A|B) = P(A) \text{ independent}$$

For Table III:

$$P(A|B) = \frac{20}{57}$$
$$P(A) = \frac{53}{100}$$
$$P(A|B) \neq P(A) \text{ dependent}$$

12. Which of the following functions has a graph with no vertical asymptote?

I	II	III
$f(x) = \dfrac{x^2+3x+2}{x^2+1}$	$f(x) = \dfrac{x^2+8x+15}{x+3}$	$f(x) = \dfrac{x^2+5x+6}{x-3}$

(A) I only (B) II only (C) III only *(D) Both I and II* (E) None of the answers (A) through (D) is correct.

Solution: The denominator of (A) is never 0. The numerator of (B) is $(x+3)(x+5)$, so $x=-3$ is a hole in the graph. The numerator of (C) is $(x+2)(x+3)$, so that (C) has a vertical asymptote at $x=3$.

13. Suppose an Easter basket contains 2 peeps, 5 candy eggs, and 2 mini peanut butter cups. Jacob randomly picks one piece of candy from the basket and eats it. Wilhelm then randomly picks one piece of the remaining candy and eats it. What is the probability that at least one of them ate a candy egg?

(A) $\frac{5}{6}$ (B) $\frac{5}{9}$ (C) $\frac{4}{9}$ (D) $\frac{1}{6}$ (E) None of the answers (A) through (D) is correct.

Solution: The probability that at least one of Jacob or Wilhelm eat a candy egg is the same as 1 minus the probability that neither of them eat a candy egg:

$$\begin{aligned} P(\text{at least 1}) &= 1 - P(\text{neither}) \\ &= 1 - \frac{4}{9}\left(\frac{3}{8}\right) \\ &= 1 - \frac{1}{6} \\ &= \frac{5}{6} \end{aligned}$$

14. Find the equation of the line that connects the vertex of the parabola $y = x^2 - 4x + 8$ with the center of the circle $(x-5)^2 + (y+3)^2 = 4$.

(A) $y = \frac{4}{3}x + \frac{32}{3}$ (B) $y = \frac{-1}{2}x - \frac{13}{2}$ *(C) $y = \frac{-7}{3}x + \frac{26}{3}$* (D) $y = \frac{5}{2}x + \frac{23}{4}$
(E) None of the answers (A) through (D) is correct.

Solution: Completing the square for the parabola, $y = (x^2 - 4x + 4) + 8 - 4 = (x-2)^2 + 4$, so that the vertex is at the point $(2, 4)$. The center of the circle is the point $(5, -3)$. The slope of a line connecting these points is $m = \frac{-3-4}{5-2} = \frac{-7}{3}$. Using the point-slope form of the line, $y - 4 = \frac{-7}{3}(x-2) = \frac{-7}{3}x + \frac{14}{3}$, so that the equation of the line is $y = \frac{-7}{3}x + \frac{26}{3}$.

15. What is the value of x if $3(9^{2x+1}) = \dfrac{27^{x-8}}{3^{3-5x}}$?

(A) $\dfrac{29}{4}$ (B) -4 *(C) $\dfrac{15}{2}$* (D) $\dfrac{-29}{6}$ (E) None of the answers (A) through (D) is correct.

Solution:

$$\begin{aligned} 3(9^{2x+1}) &= \frac{27^{x-8}}{3^{3-5x}} \\ 3(3^{4x+2}) &= \frac{3^{3x-24}}{3^{3-5x}} \\ 3^{4x+3} &= 3^{8x-27} \\ 4x + 3 &= 8x - 27 \\ 30 &= 4x \\ \frac{15}{2} &= x \end{aligned}$$

16. Which of the following is equivalent to $\dfrac{4x^3y^2z^5}{6xy^{7/3}z^4} \div \dfrac{3xy^{3/2}z^2}{12xy^5z^3}$.

(A) $\frac{8}{3}x^2y^{19/6}z^2$ (B) $\frac{3}{2}xy^{13/6}z^2$ (C) $\frac{1}{6}x^3y^{7/6}z^3$ (D) $\frac{5}{12}x^3y^{11/6}z^3$
(E) None of the answers (A) through (D) is correct.

Solution:

$$\begin{aligned} \frac{4x^3y^2z^5}{6xy^{7/3}z^4} \div \frac{3xy^{3/2}z^2}{12xy^5z^3} &= \frac{4x^3y^2z^5}{6xy^{7/3}z^4} \cdot \frac{12xy^5z^3}{3xy^{3/2}z^2} \\ &= \frac{2x^2z}{3y^{1/3}} \cdot \frac{4y^{7/2}z}{1} \\ &= \frac{8x^2y^{7/2}z^2}{3y^{1/3}} \\ &= \frac{8x^2y^{21/6}z^2}{3y^{2/6}} \end{aligned}$$

17. Which of the following solutions is equivalent to $(4+3i)^2 - \left(\frac{2}{1-i}\right)$

(A) $24+23i$ (B) $6+11i$ (C) $5+26i$ *(D) $6+23i$*
(E) None of the answers (A) through (D) is correct.

Solution:

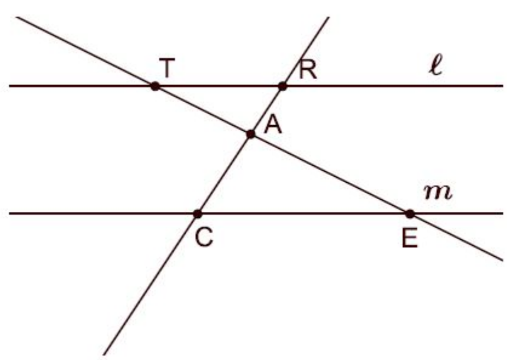
$$\begin{aligned}(4+3i)^2 - \left(\frac{2}{1-i}\right) &= (16+12i+12i+9i^2) - \left(\frac{2}{1-i}\right)\\ &= (7+24i) - \left(\frac{2}{1-i}\right)\\ &= (7+24i) - \left(\frac{2}{1-i}\right)\left(\frac{1+i}{1+i}\right)\\ &= (7+24i) - \frac{2(1+i)}{2}\\ &= (7+24i) - (1+i)\\ &= 6+23i\end{aligned}$$

18. Solve the equation $\log_4(x+3) = 16$.

(A) 1 (B) 2 (C) -1 (D) 9 *(E) None of the answers (A) through (D) is correct.*

Solution: This equation is equivalent to $4^{16} = x+3$. The number $-3+4^{16}$ is much bigger than any of the options provided.

19. In the figure shown below, lines ℓ and m are parallel. If the length of $\overline{TA} = 3$, the length of $\overline{TE} = 12$, and the length of $\overline{RA} = 2$, what is the length of $\overline{AC}$?



(A) 4 *(B) 6* (C) 8 (D) 10 (E) None of the answers (A) through (D) is correct.

Solution: Using $\overline{TA}$ and $\overline{TE}$, we find $\overline{AE} = 9$. Note that triangle TRA and triangle ACE are similar triangles (using properties of parallel lines etc.). Therefore, the ratio of sides are equivalent to each other:

$$\frac{\overline{TA}}{\overline{AE}} = \frac{\overline{RA}}{\overline{AC}}$$
$$\frac{3}{9} = \frac{2}{\overline{AC}}$$
$$3\overline{AC} = 18$$
$$\overline{AC} = 6$$

20. The right triangle ABC has sides of the following lengths:

AB=24, BC=7, AC=25

Let there exist a point D so that D is the midpoint of AB. What is the length of CD?

(A) $\sqrt{139}$ *(B) $\sqrt{193}$* (C) 12 (D) 16 (E) None of the answers (A) through (D) is correct.

Solution: The given information leads to the creation of a right triangle DBC, with sides DB=12, and BC=7. Using the Pythagorean theorem, we get:

$$CD^2 = 7^2 + 12^2$$
$$CD^2 = 193$$
$$CD = \sqrt{193}$$

21. The equation $\frac{x^2}{4} - \frac{y^2}{36} = 2$ has a graph that is symmetric with respect to the:

(A) x-axis (B) y-axis (C) x-axis and y-axis *(D) x-axis, y-axis, and the origin* (E) None of the answers (A) through (D) is correct.

Solution: Checking symmetry: x-axis replace (x, y) by $(x, -y)$, y-axis replace (x, y) by $(-x, y)$, origin replace (x, y) by $(-x, -y)$. All of these result in equivalent equations.

22. For what value of k will the system of equations

$$2x - 3y = -4$$
$$-3x + ky = 6$$

have an infinite number of solutions?

(A) 9/2 (B) 2 (C) 4/3 (D) 1 (E) None of the answers (A) through (D) is correct.

Solution: Multiplying the first equation by 3, we get $6x - 9y = -12$. Multiplying the second equation by -2, we get $6x - 2ky = -12$. There are infinite solutions if $9 = 2k$, so that $k = 9/2$.

23. A fifth-degree polynomial function with real coefficients has five zeros. It is known that three of them are 3, i, and $1+i$. What is the product of all five zeros?

(A) i (B) 0 *(C) 6* (D) -6 (E) None of the answers (A) through (D) is correct.

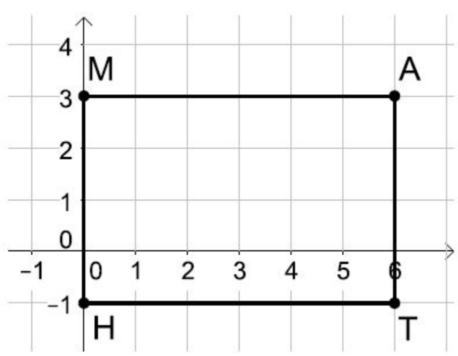
Solution: Since the polynomial has real coefficients, the complex roots show up with their conjugates, so the roots are 3, i, $-i$, $1+i$, $1-i$. The product is: $3(i)(-i)(1+i)(1-i) = 3(-i^2)(1-i^2) = 3(1)(1+1) = 6$.

24. Which of the following statements is true?

(A) A second-degree polynomial with real coefficients has two real roots (not necessarily distinct).
(B) A function cannot have more than one horizontal asymptote.
(C) Doubling the radius of circle will double its circumference.
(D) If the domain of a function $f(x)$ is $\mathbb{R}$, then it has an inverse $f^{-1}(x)$ with domain $\mathbb{R}$.
(E) None of the answers (A) through (D) are true.

Solution: (A): they could be complex conjugates; (B) a function could approach different values as $x \to \pm\infty$; (c) circumference is $2\pi r$, and $2\pi(2r) = 2[2\pi r]$ so this statement is true; (D) if it doesn't pass the horizontal line test, it won't have an inverse, and even if it does, the domain may not be the same (ex. $f(x) = e^x$, then $f^{-1}(x) = \ln(x)$)

25. The rectangle shown below has vertices M at (0,3), A at (6,3), T at (6,-1), and H at (0,-1). If the center of dilation is the origin, dilate the rectangle using a scale factor of $\frac{1}{2}$, then rotate the resulting figure counterclockwise by 90°. What is the sum of the y-coordinates of vertices A and T after applying the transformation described here?



(A) 2 (B) 3 (C) -1 (D) 1 *(E) None of the answers (A) through (D) is correct.*

Solution: The table below shows how the vertices change:

Original Point	Dilation	Rotation
M (0,3)	M (0,3/2)	M (-3/2,0)
A (6,3)	A (3,3/2)	A (-3/2,3)
T (6,-1)	T (3,-1/2)	T (1/2,3)
H (0,-1)	H (0,-1/2)	H (1/2,0)

The sum of the y-coordinates of A and T is 6, which is not one of the given choices.

26. Mike has 10 coins in his pocket. Some are quarters, some are dimes, and the rest are nickels. Aaron has twice as many quarters as Mike and half as many dimes, but the same number of nickels. Scott has the same number of quarters and dimes as Mike, but no nickels. If Aaron has \$2.75, and Scott has \$1.45, how much money does Mike have in his pocket?

(A) \$1.60 (B) \$1.85 (C) \$1.35 (D) \$1.50
(E) None of the answers (A) through (D) is correct.

Solution: This application describes the system

$$q + d + n = 10$$
$$50q + 5d + 5n = 275$$
$$25q + 10d = 145.$$

The third equation can be solved for d, resulting in $d = 14.5 - 2.5q$. Plugging this into the first equation results in $q + 14.5 - 2.5q + n = 10$, which can be solved for n, resulting in $n = -4.5 + 1.5q$. Pluging both of these into the second equation, $50q + 5(14.5 - 2.5q) + 5(-4.5 + 1.5q) = 275$, which can be solved for q to find $q = 5$. Plugging this into the previous equations for d and n results in $d = 2$ and $n = 3$. These coins correspond to \$1.60.

27. If square $ABCD$ has the same area as a circle of radius 4, what is the length of line segment AC?

(A) 8π *(B)* $4\sqrt{2\pi}$ (C) $8\sqrt{\pi}$ (D) $\sqrt{8\pi}$ (E) None of the answers (A) through (D) is correct.

Solution: The area of the circle is 16π. A square with this area would have side length of $4\sqrt{\pi}$. The diagonal of this square would have length $4\sqrt{2}\sqrt{\pi}$ by the Pythagorean Theorem.

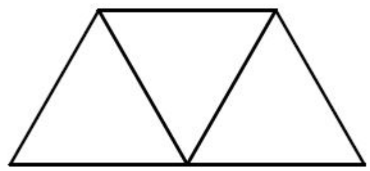
28. Which of the following is equivalent to the statement: $\log\left(\frac{(x+2)^3}{x^5(2x-3)^7}\right)$?

(A) $-2\log(x) + 3\log(2) - 7\log(2x) + 7\log(3)$
(B) $-2\log(x) + 3\log(2) + 7\log(2x) - 7\log(3)$
(C) $3\log(x+2) - 5\log(x) - 7\log(2x-3)$
(D) $3\log(x+2) - 5\log(x) + 7\log(2x-3)$
(E) None of the answers (A) through (D) is correct.

Solution: Using logarithm properties,

$$\begin{aligned}\log\left(\frac{(x+2)^3}{x^5(2x-3)^7}\right) &= \log((x+2)^3) - \log(x^5(2x-3)^7)\\ &= \log((x+2)^3) - \log(x^5) - \log((2x-3)^7)\\ &= 3\log(x+2) - 5\log(x) - 7\log(2x-3).\end{aligned}$$

29. Suppose 3 equilateral triangles are placed as shown in the image below:



If the outer perimeter of the object shown above is 40 feet, what is the area of the object?

(A) $16\sqrt{3}$ ft^2 *(B) $48\sqrt{3}$ ft^2*   (C) $32\sqrt{5}$ ft^2 (D) $96\sqrt{5}$ ft^2
(E) None of the answers (A) through (D) is correct.

Solution: Sicne the triangles are equilateral, all sides of each triangle are the same length. So

$$\begin{aligned}40 &= 5x\\ 8 &= x\end{aligned}$$

Using the length above, we can determine the height of each triangle, using the Pythagorean theorem:

$$\begin{aligned}64 &= h^2 + 16\\ 48 &= h^2\\ 4\sqrt{3} &= h\end{aligned}$$

Using the height, and a side, we can find the area one triangle, then multitply by 3 to get the total area.

$$\begin{aligned}A &= 3\left(\frac{1}{2}(8)(4\sqrt{3})\right)\\ &= 48\sqrt{3}\end{aligned}$$

30. Suppose $f(x) = -5x + 2$ and $g(x) = 50x^2 - 6$. What is the value of $\left(f^{-1} \circ g\right)(x)$?

(A) $-10x^2 + 4/5$ (B) $10x^2 - 8/5$ (C) $\dfrac{1}{-250x^2 + 32}$ *(D) $-10x^2 + 8/5$*
(E) None of the answers (A) through (D) is correct.

Solution: First we need to obtain f^{-1}:

$$\begin{aligned} y &= -5x + 2 \\ y - 2 &= -5x \\ \frac{y-2}{-5} &= x \end{aligned}$$

Now we can calculate the function composition:

$$\begin{aligned} f^{-1} \circ g(x) &= \frac{(50x^2 - 6) - 2}{-5} \\ &= -10x^2 + 8/5 \end{aligned}$$

31. Suppose A varies jointly with $\frac{1}{r-3}$ and t^3. When $r = 6$ and $t = 3$, $A = 18$. What is A if $r = 11$ and $t = 4$?

(A) 16 (B) 72 (C) $\frac{16}{9}$ (D) 36 (E) None of the answers (A) through (D) is correct.

Solution: First we need to find the constant of variation:

$$\begin{aligned} A &= k\left(\frac{t^3}{r-3}\right) \\ 18 &= k\left(\frac{3^3}{6-3}\right) \\ 18 &= k(9) \\ 2 &= k \end{aligned}$$

Using the constant of variation, we can solve the question as follows:

$$\begin{aligned} A &= 2\left(\frac{t^3}{r-3}\right) \\ &= 2\left(\frac{4^3}{11-3}\right) \\ &= 2(8) \\ &= 16 \end{aligned}$$

32. Two circles of different radii are drawn such that they have the same center point. A chord of the larger circle is drawn tangent to the smaller circle and measures 20 inches. What is the area bounded between the two circles?

(A) 25π in^2 (B) 50π in^2 *(C) 100π in^2* (D) 200π in^2
(E) None of the answers (A) through (D) is correct.

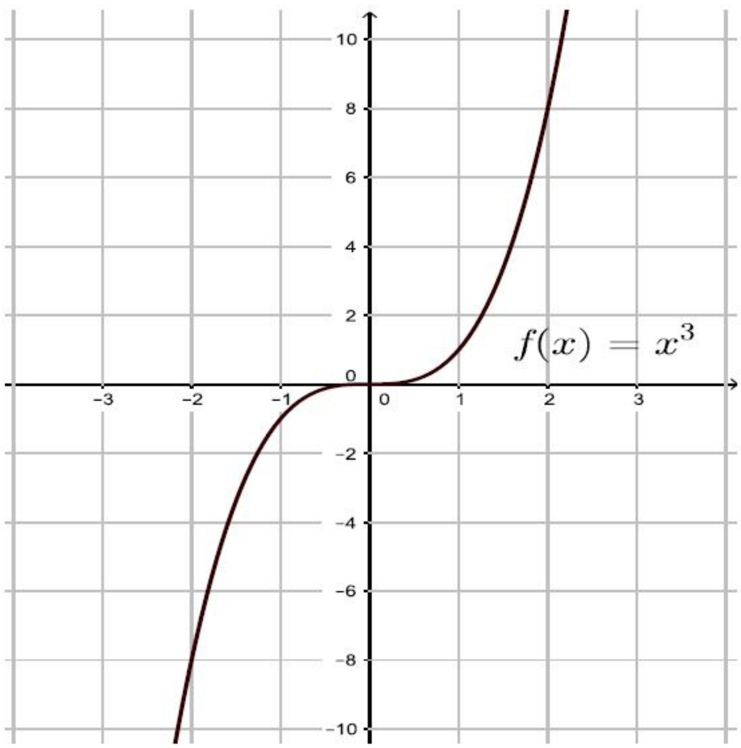
Solution: When the chord is drawn, a right triangle can be created with 1 leg equal to 10 inches, one leg is the radius of the inner circle, r_1 and the hypotenuse is the radius of the outer circle, r_2. The area bound between the two circles is $\pi r_1^2 - \pi r_2^2$ in^2. Using the Pythagorean theorem, we can rewrite the value of r_2 in terms of r_1:

$$r_1^2 = r_2^2 + 10^2$$
$$r_1^2 - 100 = r_2^2$$
$$\sqrt{r_1^2 - 100}$$

By substitution, we can simplify the statement $\pi r_1^2 - \pi r_2^2$:

$$\begin{aligned} \pi r_1^2 - \pi r_2^2 \text{ in}^2 &= \pi r_1^2 - \pi(\sqrt{r_1^2 - 100})^2 \text{ in}^2 \\ &= \pi r_1^2 - \pi(r_1^2 - 100) \text{ in}^2 \\ &= \pi r_1^2 - \pi r_1^2 + \pi 100 \text{ in}^2 \\ &= \pi 100 \text{ in}^2 \end{aligned}$$

33. The graph of $f(x) = x^3$ is illustrated below:



If $g(x) = f(x+1) + 2$, then what is $g^{-1}(10)$?

(A) 0 *(B) 1* (C) 2 (D) 3 (E) None of the answers (A) through (D) are true.

Solution: If we shift the graph one left and two up, which x-value corresponds to $y = 10$? It will be $x = 1$. (Just follow what happens to $y = 8$ as you shift the graph.) This could also be calculated algebraically.

34. If $a + b = \frac{1}{2}$ and $a^2 + b^2 = 1$, then what does $a^3 + b^3$ equal?

(A) 7 (B) $\frac{1}{8}$ (C) $\frac{-3}{8}$ *(D)* $\frac{11}{16}$ (E) None of the answers (A) through (D) is correct.

Solution: First find the values of a and b:

$$\begin{aligned}
(a+b)^2 &= \left(\frac{1}{2}\right)^2 \\
a^2 + 2ab + b^2 &= \frac{1}{4} \\
1 + 2ab &= \frac{1}{4} \\
2ab &= \frac{-3}{4} \\
ab &= \frac{-3}{8}
\end{aligned}$$

Using the equation above, we can obtain the solution:

$$\begin{aligned}
a^3 + b^3 &= (a+b)(a^2 - ab + b^2) \\
&= \left(\frac{1}{2}\right)\left(1 - \frac{-3}{8}\right) \\
&= \left(\frac{1}{2}\right)\left(\frac{11}{8}\right) \\
&= \frac{11}{16}
\end{aligned}$$