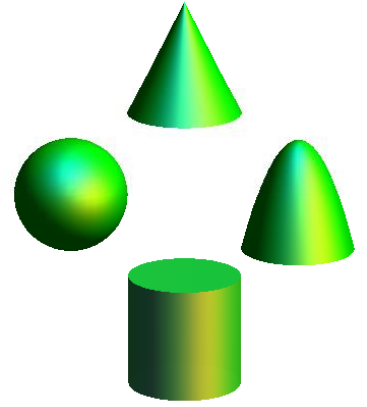




2015
High School Math Contest

Level 2
Exam



Lenoir-Rhyne University
*Donald and Helen Schort School of
Mathematics and Computing Sciences*

This exam has been prepared by the following faculty from **Western Carolina University**:

John Wagaman, Chairperson

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Do's and Don'ts:

- **Do NOT** open this booklet until you are instructed to do so.
- **NO Calculators (Or other electronic devices)**
- Contestants with electronic devices (on or off) will be **disqualified!**
And their **team** will be **disqualified!**
(The other team members will continue to participate in the individual contest.)

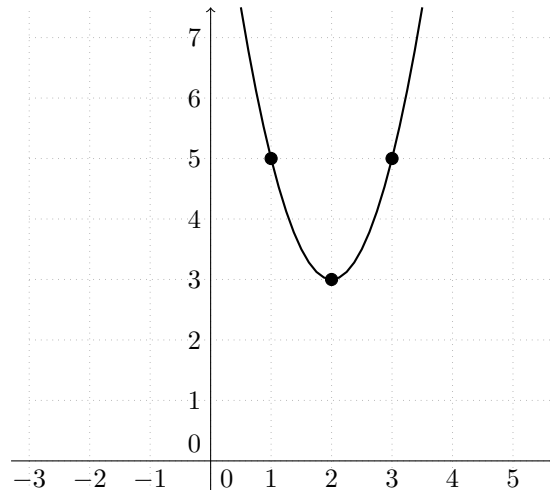
DIRECTIONS:

- **The Score Card:**
Write:
 - Your **NAME** on the “name line” (of course).
 - “**Level 3**” on the “subject line”.
 - Your **SCHOOL** on the “**Date Line**”
- **Clearly mark** ONE bubble using **#2 PENCIL**.
 - Light marks will be counted as unmarked!
 - **Completely erase** any changes.
- You **can** write on this test booklet. (But the test booklet will not be graded.)
- **Tie Breakers:** In case of ties, the person with the least number of *wrong answers* wins.
(A *blank* is better than *incorrect!*)
- **The Exam:** 34 problems, 70 minutes.

- **WAIT** for the signal to begin.

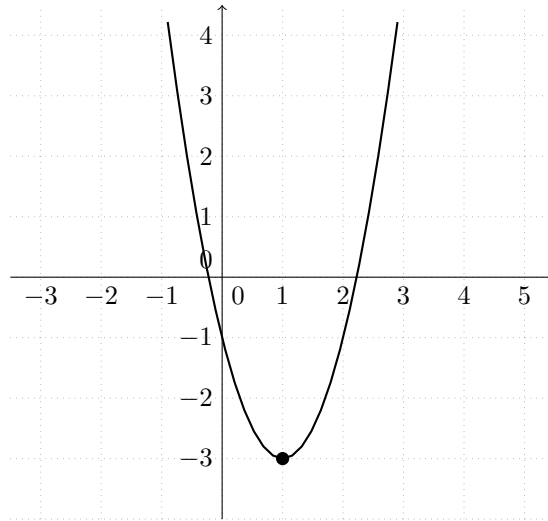
1. A circle is unwound and re-shaped into a square. Find the ratio of the square's area to the circle's area.
- (A) $\frac{\pi}{8}$ (B) $\frac{\pi}{4}$ (C) $\frac{1}{2}$ (D) $\frac{1}{\sqrt{2}}$ (E) $\frac{4}{\pi}$
2. Given $f(x) = x + 2$ and $g(x) = \sqrt[3]{x}$, find $f^{-1}(g^{-1}(2))$.
- (A) 8 (B) 6 (C) 2 (D) -2 (E) -6
3. An isosceles triangle $\triangle ABC$ has a smallest angle A that is one-third of the size of the smallest of the remaining angles — (that is, the smallest of B and C). What is the approximate size of angle A ?
- (A) 23.0° (B) 25.7° (C) 45° (D) 77.1° (E) None of the answers (A)–(D) is correct.
4. One solution of $x^3 + 5x^2 - 2x - 4 = 0$ is $x = 1$. Which of the following is another solution?
- (A) $-1 + \sqrt{7}$ (B) $-3 + \sqrt{5}$ (C) $-2 + \sqrt{5}$ (D) $-3 + \sqrt{3}$ (E) $-5 + \sqrt{2}$
5. If a , b , and c are real numbers such that $\frac{a}{b} = 3$ and $\frac{b}{c} = 7$, then $\frac{a+b}{b+c}$ equals:
- (A) $\frac{7}{2}$ (B) $\frac{7}{8}$ (C) $\frac{3}{7}$ (D) $\frac{1}{7}$ (E) 21
6. Find the sum of all values of m that make the polynomial $x^2 + (m + 5)x + (5m + 1)$ a perfect square.
- (A) 3 (B) 4 (C) 7 (D) 8 (E) 10
7. Find the sum of all real values of x that satisfy the equation $||x - 2| - 3| = 4$.
- (A) -8 (B) -4 (C) 4 (D) 8 (E) None of the answers (A)–(D) is correct.
8. Suppose a and b are the solutions to $9y^2 + 9y = 4$. What is $2|a - b| + |ab|$?
- (A) $\frac{14}{3}$ (B) $-\frac{34}{9}$ (C) $-\frac{14}{3}$ (D) $\frac{34}{9}$ (E) None of the answers (A)–(D) is correct.

9. To get from point A to point B you must avoid walking through a swamp. To avoid the swamp, you must walk 34 meters south and 41 meters east. To the nearest meter, how many meters would be saved if it were possible to walk through the swamp?
- (A) 22 (B) 34 (C) 53 (D) 75 (E) None of the answers (A)–(D) is correct.
10. Suppose $h(x) = x^2 - 2x$. What is an equation of the secant line containing $(2, h(2))$ and $(4, h(4))$?
- (A) $y = 4x - 8$ (B) $y = 10x - 16$ (C) $y = 4x$ (D) $y = 16x$
- (E) None of the answers (A)–(D) is correct.
11. Which of the following polynomials has roots 4, -3 , and -1 ?
- (A) $y = x^3 - 8x^2 + 19x - 12$ (B) $y = x^3 - 13x - 12$ (C) $y = x^3 + 8x^2 + 19x + 12$ (D) $y = x^3 - 13x + 12$
- (E) None of the answers (A)–(D) is correct.
12. Suppose $f(x) = x^2$. Below is a graph of $g(x)$, which involves transformations of $f(x)$. Using the graph below, what is $g(x)$ in terms of transforming $f(x)$?



- (A) $2f(x - 2) + 3$ (B) $\frac{1}{2}f(x + 2) + 3$ (C) $2f(x + 2) + 3$ (D) $\frac{1}{2}f(x - 2) + 3$
- (E) None of the answers (A)–(D) is correct.
13. What is the sum of the solutions of $\sqrt{3x + 1} - \sqrt{x - 1} = 2$?
- (A) 1 (B) 5 (C) 6 (D) 7 (E) None of the answers (A)–(D) is correct.

14. Consider the two quadratic functions $f(x)$ and $g(x)$, where $f(x)$ is shown below and $g(x) = x^2 + 2x - 8$. What is the y -coordinate of the lower minimum of the two functions?



- (A) 1 (B) -1 (C) -3 (D) -12 (E) None of the answers (A)–(D) is correct.
15. Real estate ads suggest that 53% of homes for sale have garages, 25% have swimming pools, and 4% have both features. Let a denote the percentage that have a garage or swimming pool or both. Let b denote the percentage that have a garage but not a pool. Let c denote the percentage that have a pool but not a garage. Find $b + c - a$.
- (A) -4% (B) 0% (C) 4% (D) 8% (E) None of the answers (A)–(D) is correct.
16. Given that $5^{x+1} = 30$, what is the value of 5^{3x+1} ?
- (A) $\log_5 30 - 1$ (B) 90 (C) 180 (D) 1080 (E) None of the answers (A)–(D) is correct.
17. Three concentric circles have radii 2, 3, and 4 inches, respectively. What percent of the area of the largest circle is the area of the middle ring, rounded to the nearest whole percent?
- (A) 12.5% (B) 25% (C) 31% (D) 33% (E) None of the answers (A)–(D) is correct.
18. Nick is a runner, and his goal is to complete four laps around a circuit at an average speed of 10 mph. If he completes the first three laps at a constant speed of only 9 mph, what speed does he need to maintain in miles per hour on the fourth lap to achieve his goal?
- (A) 11 (B) 12 (C) 13 (D) 14 (E) None of the answers (A)–(D) is correct.

19. A basketball coach has 6 players to assign to 5 different positions (center, power forward, small forward, shooting guard, point guard) on the court, and one will sit on the bench. How many different lineups can the coach make?
- (A) 6 (B) 20 (C) 30 (D) 120 (E) None of the above are true.
20. Which of the following expressions is equivalent to $\frac{\sqrt{x^2 + 1} - x \left(\frac{2x}{2\sqrt{x^2 + 1}} \right)}{x^2 + 1}$?
- (A) $(2x^2 + 1)(x^2 + 1)^{1/2}$ (B) $\frac{1}{(x^2 + 1)^{3/2}}$ (C) $(x^2 + 1)^{1/2}$ (D) $\frac{2x^2 + 1}{(x^2 + 1)^{3/2}}$
- (E) None of the above are true.
21. What is the digit in the ones place of 2^{57} ?
- (A) 2 (B) 4 (C) 6 (D) 8 (E) None of the answers (A)–(D) is correct.
22. A rectangle has sides of 10 centimeters and 8 centimeters. A second rectangle with sides of 7 centimeters and 5 centimeters overlaps the first rectangle such that the overlapping section forms a square. What is the absolute value of the difference between the two non-overlapping regions of the rectangles in square centimeters?
- (A) 0 (B) 35 (C) 45 (D) 80 (E) None of the answers (A)–(D) is correct.
23. A tree has 10 pounds of apples at dawn. Every afternoon, a bird comes and eats x pounds of apples. Overnight, the amount of food on the tree increases by 10%. What is the maximum value of x such that the bird can sustain itself indefinitely on the tree without the tree running out of food?
- (A) 10/11 (B) 9/10 (C) 8/9 (D) 7/8 (E) None of the answers (A)–(D) is correct.
24. An airplane flies due north from Ft. Myers to Sarasota, a distance of 150 miles, and then turns through an angle of 50° and flies to Orlando, a distance of 100 miles. How far is it directly from Ft. Myers to Orlando?
- (A) $\sqrt{32500 - 30000 \cos(130^\circ)}$ (B) $\sqrt{32500 - 30000 \sin(50^\circ)}$ (C) $\sqrt{32500 - 30000 \cos(50^\circ)}$
- (D) $\sqrt{32500 - 30000 \sin(130^\circ)}$ (E) None of the answers (A)–(D) is correct.
25. Suppose $2x + 10 = 3$ and $4x^2 - 25y^2 = 24$. Which of the following could be a value of $2x + y$?
- (A) -6 (B) $-189/5$ (C) 0 (D) 54 (E) None of the answers (A)–(D) is correct.

26. Find the equation of the line perpendicular to the line $2x + 4y = -3$ that passes through the center of the circle $x^2 + 6x + y^2 - 8y = 36$.
- (A) $y = -\frac{1}{2}x - \frac{5}{2}$ (B) $y = 2x + 10$ (C) $y = 2x - 10$ (D) $y = -\frac{1}{2}x + \frac{5}{2}$
- (E) None of the answers (A)–(D) is correct.
27. One of Jason and Jerry always lies on Mondays, Tuesdays and Wednesdays, and always tells the truth on the other days of the week. The other always lies on Thursdays, Fridays, and Saturdays, and always tells the truth the other days of the week. At noon, the two had the following conversation:
- Jerry:** I lie on Saturdays.
- Jason:** I will lie tomorrow.
- Jerry:** I lie on Sundays.
- This conversation takes place on a:
- (A) Monday (B) Tuesday (C) Wednesday (D) Thursday (E) Friday
28. The ratio of the dimensions of a rectangular solid is 3:3:1, and the ratio of its volume to surface area is 3:1. Find the volume of the solid given that the dimensions are measured in meters.
- (A) 9000 m^3 (B) 243000 m^3 (C) 27000 m^3 (D) 3000 m^3
- (E) None of the answers (A)–(D) is correct.
29. I want to triple the volume in a cylindrical can. Due to packaging limitations, I can only increase the height of the can by 50%. Assuming I do this, approximate the percentage that the radius of the can must be increased to triple the volume.
- (A) 41.4% (B) 45.7% (C) 52.1% (D) 141.4% (E) None of the answers (A)–(D) is correct.
30. Suppose you are driving a car with 20 inch diameter tires and are traveling down the turnpike at a constant speed of 30π mph. If you have been traveling under these conditions for 30 minutes, how many complete revolutions did your tires make?
- (A) 15,840 (B) 31,680 (C) 47,520 (D) 63,360 (E) None of the answers (A)–(D) is correct.
31. Suppose you start with a right triangle, and then you increase its base by 5 inches and decrease its height by 7 inches. If the new shape is an isosceles right triangle with the same area as the original, what is the area (in square inches)?
- (A) 153.125 (B) 167.500 (C) 306.250 (D) 335.001
- (E) None of the answers (A)–(D) is correct.

32. A motorboat can maintain a constant speed of 16 miles per hour relative to the water. The boat makes a trip upstream to a certain point in 20 minutes; the return trip takes 15 minutes. What is the speed of the current?

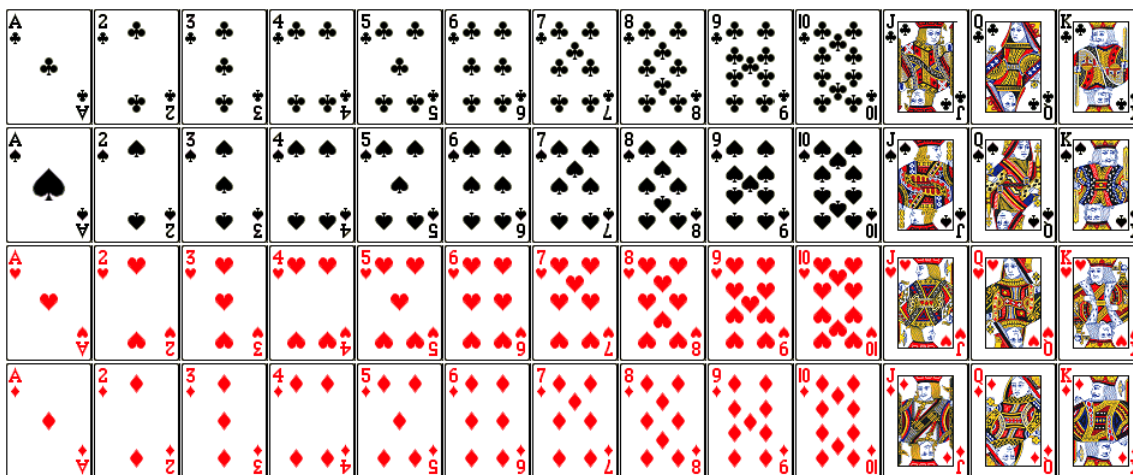
(A) 1.7 mph (B) 2.3 mph (C) -1.7 mph (D) 0.6 mph

(E) None of the answers (A)–(D) is correct.

33. Given that $f(x) + 2f(4 - x) = x + 8$, compute $f(16)$.

(A) $-32/3$ (B) $9/10$ (C) $8/9$ (D) $7/8$ (E) None of the answers (A)–(D) is correct.

34. A deck of playing cards contains 52 cards, of which 26 are black and 26 are red. The 26 black cards are divided into two suits of 13 spades and 13 clubs. The 26 red cards are divided into two suits of 13 hearts and 13 diamonds. Each of the suits has cards of ranks Ace, 2, 3, 4, 5, 6, 7, 8, 9, 10, Jack, Queen and King. The ranks of Jack, Queen and King are considered face cards. Let F denote the set of face cards, let D denote the set of diamonds, let R denote the set of red cards and E the set of even-numbered ranks (2, 4, 6, 8, 10). The image below shows each of the cards in a standard deck. Which of the following statements are true?



(A) F and D are independent. (B) R and D are independent. (C) E and F are independent.

(D) Exactly two of the above are true. (E) None of the above are true.

