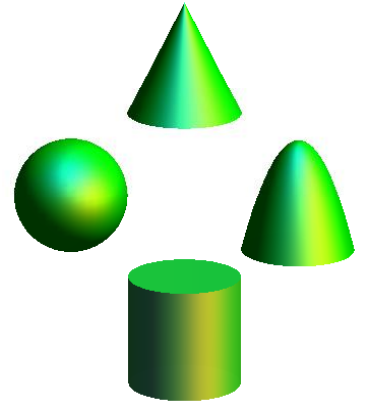




2014
High School Math Contest

Level II
Math 2 & Math 3
Exam



Lenoir-Rhyne University

Donald and Helen Schort School of

This exam has been prepared by the following faculty from **Western Carolina University**:

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Do's and Don'ts:

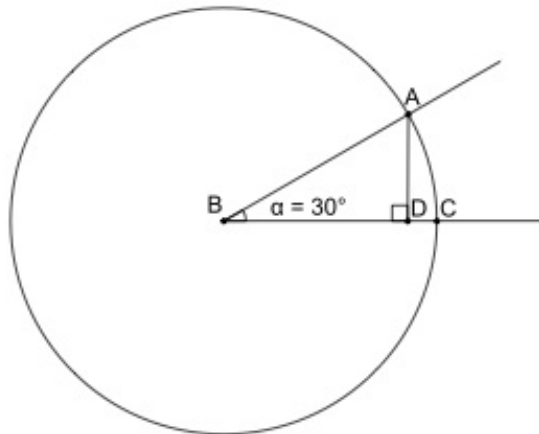
- **Do NOT** open this booklet until you are instructed to do so.
- **NO Calculators** (Or other electronic devices)
- Contestants with electronic devices (on or off) will be **disqualified!**
And their **team** will be **disqualified!**
(The other team members will continue to participate in the individual contest.)

DIRECTIONS:

- **The Score Card:**
Write:
 - Your **NAME** on the “name line” (of course).
 - “**MATH 2**” on the “subject line”.
 - Your **SCHOOL** on the “Date Line”
 - **Clearly mark** ONE bubble using **#2 PENCIL**.
 - Light marks will be counted as unmarked!
 - **Completely erase** any changes.
 - You **can** write on this test booklet. (But the test booklet will not be graded.)
 - **Tie Breakers:** In case of ties, the person with the least number of *wrong answers* wins.
(A *blank* is better than *incorrect!*)
 - **The Exam:** 30 problems, 70 minutes.
- **WAIT** for the signal to begin.

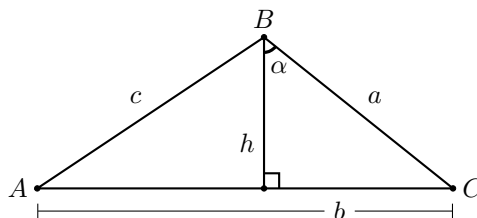
1. Find the equation of the line perpendicular to the line $3x - 2y = 4$ that passes through the vertex of the parabola $y = x^2 + 6x + 16$.
- (A) $y = \frac{2}{3}x + 9$ (B) $y = -\frac{3}{2}x + \frac{5}{2}$ (C) $y = -\frac{2}{3}x + 5$ (D) $y = \frac{3}{2}x + \frac{23}{2}$ (E) $y = -\frac{2}{3}x + 7$
2. Solve $\frac{2x}{x^2 - 4} = \frac{4}{x^2 - 4} - \frac{3}{x + 2}$ for x .
- (A) 2 (B) $-\frac{2}{5}$ (C) -2 (D) $\frac{2}{5}$ (E) None of the answers (A)–(D)
3. An urn contains 3 red balls, 2 blue balls, and 5 green balls. Two balls are removed at random. What is the probability that one red ball and one green ball are removed?
- (A) $\frac{1}{5}$ (B) $\frac{1}{9}$ (C) $\frac{2}{3}$ (D) $\frac{1}{6}$ (E) $\frac{1}{3}$
4. A circle defined by the equation $x^2 + y^2 = 3$ is intersected by the line $2x - y = 2$. What is the product of the two x -coordinates of the points of intersection?
- (A) $\frac{8}{5}$ (B) $-\frac{1}{5}$ (C) $\frac{1}{5}$ (D) $-\frac{8}{5}$ (E) None of the answers (A)–(D)
5. You have a couch that is 6.5 feet long. You are deciding where you want to place the couch in the living room. You do not want to push the couch around, and you do not have any measuring tools. However, you have several dollar bills in your wallet, and you remember that a dollar bill is 6 inches long. How many dollar bills will it take to measure out the length of the couch?
- (A) 13 dollar bills (B) 26 dollar bills (C) 13 inches (D) 26 inches
- (E) None of the answers (A)–(D)
6. Calculate the product of the solutions to the equation $x^2 - 6x = -13$.
- (A) 5 (B) 13 (C) 10 (D) 2 (E) None of the answers (A)–(D)
7. $\frac{[(4 + 2i)(3 + i) - (5 + 2i)(5 - 2i)]i}{2 + i}$ is equivalent to:
- (A) $-\frac{39}{5} - \frac{28}{5}i$ (B) $-\frac{31}{5} - \frac{12}{5}i$ (C) $\frac{13}{5} - \frac{17}{5}i$ (D) $-\frac{1}{5} - \frac{48}{5}i$
- (E) None of the answers (A)–(D)

8. Arc AC is subtended by a central angle of 30° , \overline{AD} , which is of length 3, is perpendicular to \overline{BC} , and \overline{BD} is of length $3\sqrt{3}$ as seen in the figure below. Find the length of arc AC .



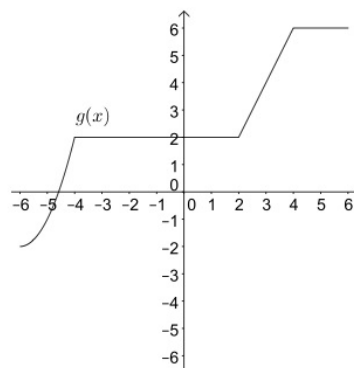
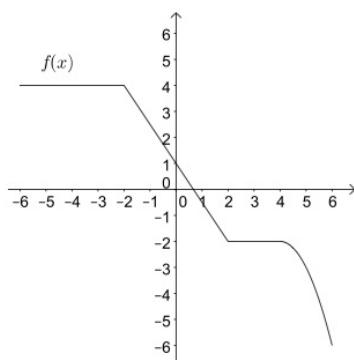
- (A) π (B) 6π (C) $\frac{\pi}{6}$ (D) 2π (E) None of the answers (A) through (D)

9. Find the area of the triangle shown below. Assume α is measured in degrees.



- (A) Area = $\frac{1}{2}ab\sin(\alpha)$ (B) Area = $\frac{1}{2}bc\sin(90^\circ - \alpha)$ (C) Area = $\frac{1}{2}ab\sin(90^\circ - \alpha)$
 (D) Area = $\frac{1}{2}ac\sin(\alpha)$ (E) None of the answers (A)–(D)
10. Suppose there are four points such that point A is at $(2, 3)$, point B at $(-4, -3)$, point C at $(4, 5)$, and point D at $(-2, 1)$. Calculate the distance between the midpoints of \overline{AB} and \overline{CD}
- (A) 13 (B) $\sqrt{61}$ (C) 61 (D) $\sqrt{13}$ (E) None of the answers (A)–(D)
11. Suppose that the number of bacteria in a bottle doubles every 5 minutes. Furthermore, suppose that a single bacterium was present at 6:00 a.m., and the bottle becomes full at 9:00 a.m. What fraction of the bottle is full at 7:15?
- (A) $\frac{1}{2^{105}}$ full (B) 2^{75} full (C) 2^{15} full (D) $\frac{1}{2^{21}}$ full (E) None of the answers (A)–(D)

12. Use the two graphs below to calculate $f(g(3)) + g(f(-4))$.



- (A) 0 (B) -2 (C) 8 (D) 6 (E) None of the answers (A)–(D)

13. For what values of k will the product of all solutions to $x^2 + kx + 9 = 0$ be nonnegative?

- (A) $k \geq 6$ (B) $k \leq -6$ and $k \geq 6$ (C) $k \geq \frac{9}{2}$ (D) $k \leq -\frac{9}{2}$ and $k \geq \frac{9}{2}$ (E) all real k

14. A polynomial with real coefficients and zeros 3, -1, and $1 + 2i$ is:

- (A) $x^4 - 2x^2 + 16x - 15$ (B) $x^4 - 4x^3 + 6x^2 - 4x - 15$ (C) $x^4 - 6x^2 - 8x - 3$
 (D) $x^4 - 2x^3 + x^2 - 8x - 12$ (E) $x^3 - 7x - 6$

15. If $x + 2$ is a factor of $x^4 + x^3 + kx^2 + 5x - 2k$, what is k ?

- (A) 1 (B) -1 (C) 17 (D) -17 (E) None of the answers (A)–(D)

16. Which of the following statements is/are true regarding the graph of $f(x) = \frac{2x^2 - 2x - 12}{3x^2 - 27}$?

- (A) There is a vertical asymptote at $x = 3$. (B) There is a vertical asymptote at $x = -3$.
 (C) There is a horizontal asymptote at $y = \frac{2}{3}$. (D) Exactly two of the previous are true.
 (E) Each of the statements in (A)–(C) is true.

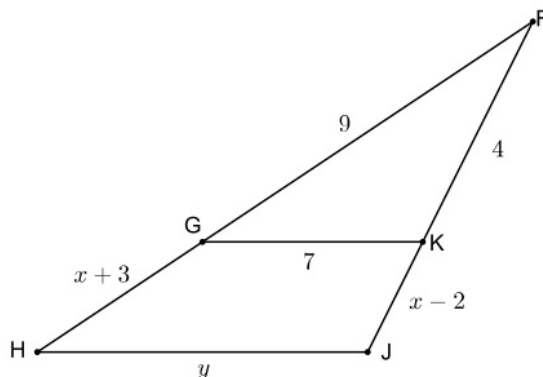
17. Solve for x : $\log_3(\log_2(x)) = 2$

- (A) 3 (B) 9 (C) 81 (D) 512 (E) 1024

18. Solve for x : $9^{3x-4} = \frac{81^{2x+1}}{\sqrt{27}}$.

- (A) $\frac{14}{5}$ (B) $-\frac{21}{4}$ (C) $-\frac{7}{3}$ (D) $-\frac{20}{3}$ (E) None of the answers (A)–(D)

19. Given $\overline{GK} \parallel \overline{HJ}$ with lengths as shown, find the perimeter of $\triangle HJF$.



- (A) 40 (B) 38 (C) 35 (D) 49 (E) 50

20. Town A and Town B are 200 miles apart. Car A leaves Town A at noon, and drives toward Town B at 40 mph. At 1pm, Car B leaves Town B at 60 mph, driving along the same road toward Town A. After driving 40 miles, Car B realizes they forgot a few things and drove back to Town B. After spending 10 minutes there, Car B gets back on the highway driving toward Town A, again at 60 mph. At what time do the cars meet?

- (A) 2:30 pm (B) 3:00 pm (C) 3:30 pm (D) 4:00 pm (E) 4:30 pm

21. Which of the following are true about the transformation of $f(x)$ to $g(x)$ if $f(x) = x^2$ and $g(x) = 2x^2 + 5x + 3$?

- (A) Shifted right by $\frac{5}{4}$ (B) Shifted down by $\frac{1}{8}$ (C) Shifted left by $\frac{5}{4}$
 (D) Both (A) and (B) are correct (E) Both (B) and (C) are correct

22. Consider a rectangular swimming pool 40 feet long and 25 feet wide. The shallow end is 3 feet deep and extends for 6 feet. Then, for 24 feet horizontally, there is a constant slope downwards to the 10 foot-deep end. One gallon of pool paint covers 80 square feet of surface. How many gallons of paint need to be purchased to ensure the entire surface can be painted?

- (A) 22 gallons (B) 24 gallons (C) 25 gallons (D) 20 gallons
 (E) None of the answers (A) through (D)

23. Jane conducted a survey of her classmates. The survey asked the students if they enjoyed swimming and if they enjoyed hiking. John wishes to create a two-way frequency table to display the information, but Jane only gave John two bits of information:

$$P(\text{Swimming or Hiking}) = \frac{10}{17} \quad \text{and} \quad P(\text{Swimming given Hiking}) = \frac{5}{7}.$$

He wants the table to look like the empty table below:

	Enjoys Swimming	Doesn't Enjoy Swimming
Enjoys Hiking		
Doesn't Enjoy Hiking		

Which of the following tables should John use?

- (A)

7	2
3	5

 (B)

5	2
3	7

 (C)

5	3
2	7

 (D)

7	3
2	5

 (E) None of the answers (A)–(D)

24. Define the functions

$$f(x) = 3x + 4$$

$$g(x) = x^2 - 6x + 8$$

$$h(x) = 5 - x$$

$$k(x) = \frac{g(x)}{3f^{-1}(x)}.$$

Suppose $x \neq 4$. Which is equivalent to $(h \circ k)(x)$?

- (A) $7 - x$ (B) $3 - x$ (C) $\frac{(x-2)(x-4)}{-3(3x+4)}$ (D) $-x^2 + 5x + 1$ (E) $x^2 - 5x - 1$

25. Suppose $f(x) = \frac{2x+1}{x-9}$ and $g(x) = x^2 - 16$. If $H(x) = f(g(x))$, what is the domain of $(H \cdot H)(x)$?

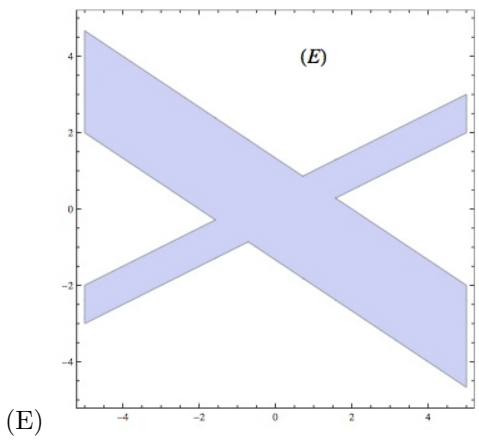
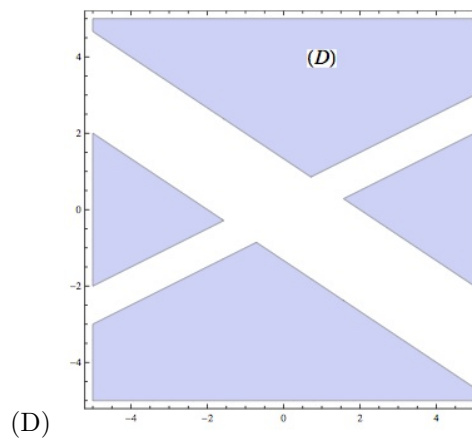
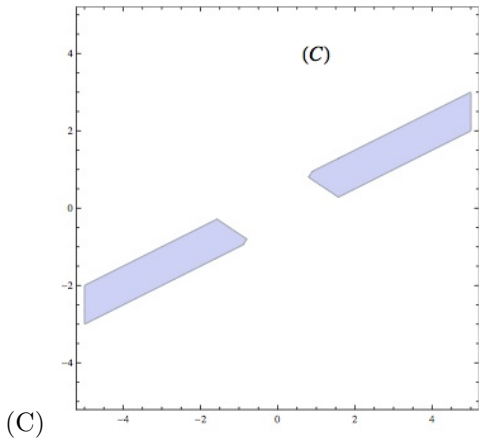
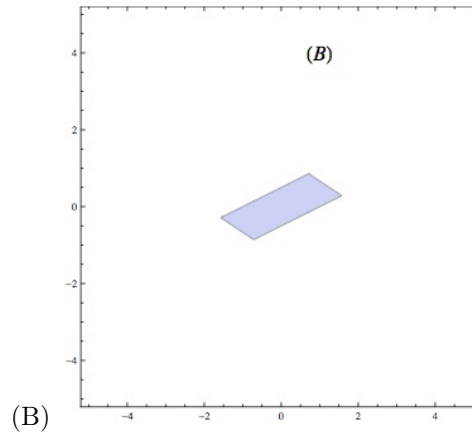
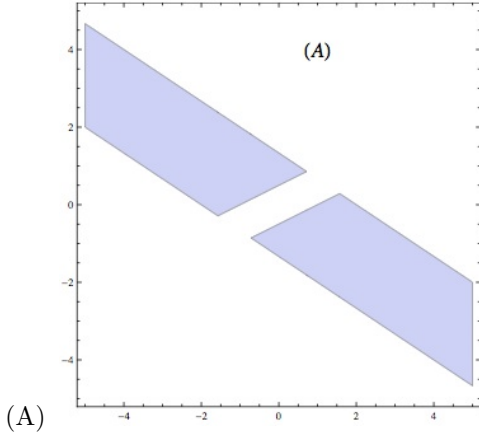
- (A) $(-\infty, 5) \cup (5, \infty)$ (B) $(-\infty, -5) \cup (-5, 5) \cup (5, \infty)$ (C) $(-\infty, -25) \cup (-25, 25) \cup (25, \infty)$
 (D) $(\infty, 4) \cup (4, \infty)$ (E) $(-\infty, -4) \cup (-4, 4) \cup (4, \infty)$

26. Suppose we start with the graph of the circle $x^2 + 4x + y^2 + 6y = 12$. We shift the graph left 7 units, then shift up 5 units, then reflect it over the x -axis. Finally, we rotate the circle 90° clockwise about the point $(-4, -2)$. What is the product of the x and y coordinates of the center of the resulting circle?

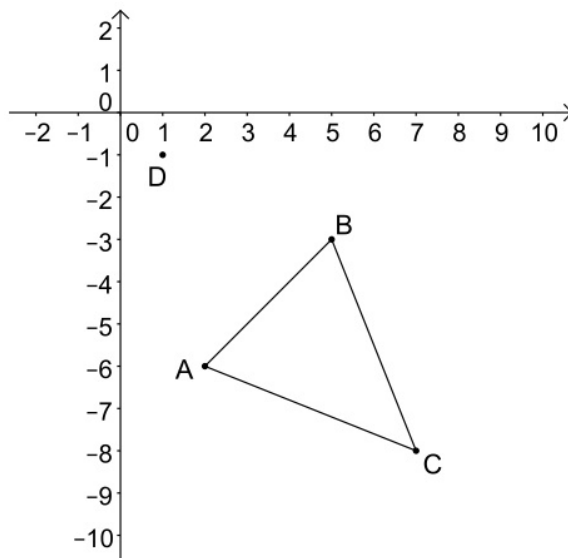
- (A) -12 (B) 12 (C) -18 (D) 18 (E) None of the answers (A)–(D)

27. Which shaded region illustrates the solutions to

$$\begin{cases} |2x + 3y| \leq 4 \\ |x - 2y| \geq 1 \\ |x| \leq 5 \\ |y| \leq 5 \end{cases}$$



28. The triangle shown below has vertices A at $(2, -6)$, B at $(5, -3)$, and C at $(7, -8)$. If the center of dilation is point D at $(1, -1)$ and the scale factor is $1/2$, calculate the product of the x -coordinates of vertices A and C after the dilation.



- (A) 12 (B) $\frac{9}{2}$ (C) 6 (D) $\frac{63}{4}$ (E) None of the answers (A)–(D)
29. Derive the equation of a parabola with focus at $(-3, 4)$ and the directrix $y = 2$.
- (A) $y = \frac{1}{4}x^2 + \frac{3}{2}x + \frac{21}{4}$ (B) $y = \frac{1}{4}x^2 - \frac{3}{4}$ (C) $y = \frac{1}{4}x^2 + \frac{21}{4}$ (D) $y = \frac{1}{4}x^2 - \frac{3}{2}x - \frac{3}{4}$
- (E) None of the answers (A)–(D)
30. A sample of concrete specimens of a certain type is selected, and the compressive strength of each specimen is determined. The mean and standard deviation are calculated as $\bar{x} = 3000$ and $s = 500$ respectively. The sample histogram is found to be well approximated by a normal curve. Using the empirical rule, approximately what percent of the sample observations are between 2500 and 4000?
- (A) 95% (B) 81.5% (C) 83.85% (D) 68% (E) None of the answers (A)–(D)