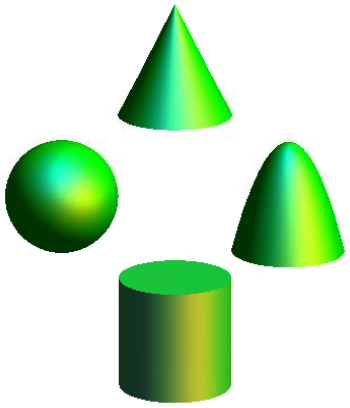
2013
High School Math Contest

Geometry Exam



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This exam has been prepared by the following faculty from **Western Carolina University**:

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Do's and Don'ts:

- **Do NOT** open this booklet until you are instructed to do so.
- **NO Calculators (Or other electronic devices)**
- Contestants with electronic devices (on or off) will be **disqualified!**
  And their **team** will be **disqualified!**
  (The other team members will continue to participate in the individual contest.)

DIRECTIONS:

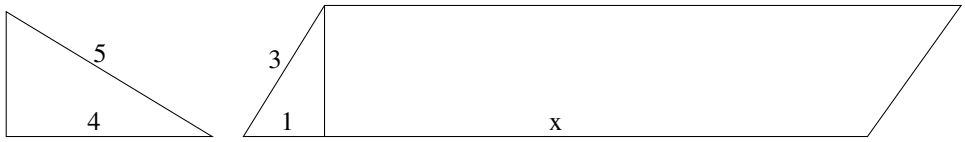
- **The Score Card:**
  - For the Geometry exam, you must use the scorecard highlighted in **GREEN**.
  - Write:
    - Your **NAME** on the "**name line**" (of course).
    - "**GEOMETRY**" on the "**subject line**".
    - Your **SCHOOL** on the "**Date Line**"
- **Clearly mark** ONE bubble using **#2 PENCIL**.
  - Light marks will be counted as unmarked!
  - **Completely erase** any changes.
- You **can** write on this test booklet. (But the test booklet will not be graded.)
- **Tie Breakers**: In case of ties, the person with the least number of *wrong answers* wins. (A ***blank*** is better than ***incorrect!***)
- **The Exam:** **25** problems, **70** minutes.

- **WAIT** for the signal to begin.

1. The equation of the perpendicular bisector of the line segment with end points $(-1, 2)$ and $(7, 8)$ is

   (A) $4y - 3x - 11 = 0$ (B) $4y + 3x - 11 = 0$ (C) $3y + 4x - 27 = 0$ (D) $3y - 4x - 27 = 0$
   (E) None of the answers (A) through (D) is correct.

2. In the figure below, if the area of the triangle is three-fifths of the area of the parallelogram then x is



   (A) $\frac{9\sqrt{2}-10}{10}$ (B) $\frac{9\sqrt{2}}{10}$ (C) $\frac{3\sqrt{2}-2}{2}$ (D) $\frac{5\sqrt{2}-2}{2}$
   (E) None of the answers (A) through (D) is correct.

3. For what real value of k will the equation $x^2 + y^2 + 2x - 4y + k$ describe a circle of radius equal to 3?

   (A) 2 (B) 8 (C) -4 (D) -9 (E) 4

4. A square and an equilateral triangle have equal perimeters. The area of the triangle is $4\sqrt{3}$ square centimeters. Expressed in centimeters, the diagonal of the square is

   (A) $4\sqrt{3}$ cm (B) $\frac{3\sqrt{2}}{2}$ cm (C) $4\sqrt{2}$ cm (D) $3\sqrt{2}$ cm
   (E) None of the answers (A) through (D) is correct.

5. The surface area of a cube is 1014 square centimeters. What is the ratio of the number of square centimeters in the surface area to the number of cubic centimeters in the volume of the cube? Express your answer as a common fraction in lowest terms.

   (A) $\frac{5}{13}$ (B) $\frac{7}{13}$ (C) $\frac{6}{13}$ (D) $\frac{4}{13}$ (E) $\frac{8}{13}$

6. A piece of cardboard is 60 centimeters by 50 centimeters. It is cut into 500 pieces to make a jigsaw puzzle. The piece of cardboard is not moved while it is being cut. The total length of cutting done is 20 meters. The average perimeter, in centimeters, of each piece is

   (A) $\frac{4}{3}$ (B) $\frac{211}{25}$ (C) $\frac{111}{25}$ (D) 8 (E) None of the answers (A) through (D) is correct.

7. A 4×6 chess board is constructed using 6 pieces, five of which are shown below. The sixth piece has the same shape as one of the following pieces (but not necessarily the same coloring). The piece whose shape appears twice is

   (A)  (B)  (C)  (D)  (E) 

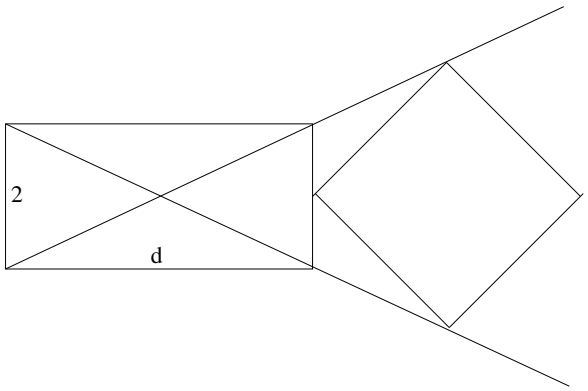
8. A circle is inscribed in an equilateral triangle, and a square is inscribed in the circle. The ratio of the area of the triangle to the area of the square is

   (A) $\sqrt{3} : 1$ (B) $\sqrt{3} : \sqrt{2}$ (C) $3\sqrt{3} : 2$ (D) $3 : \sqrt{2}$
   (E) None of the answers (A) through (D) is correct.

9. Three circles of radii 2, 4, and 6 are tangent to each other such that the center of each circle is outside of the two other circles. Find the number of square units in the area of the triangle whose vertices are the centers of the three circles.

(A) 20 (B) 22 (C) 24 (D) 26 (E) 28

10. The diagram below shows a square with one corner on a side of a $d \times 2$ rectangle, two corners on the extensions of the diagonals of that rectangle, and the fourth corner on the extension of the line joining its opposite corner to the center of the rectangle. If the square and the rectangle have the same area, determine d.



(A) 3 (B) 4 (C) 5 (D) 6 (E) None of the answers (A) through (D) is correct.

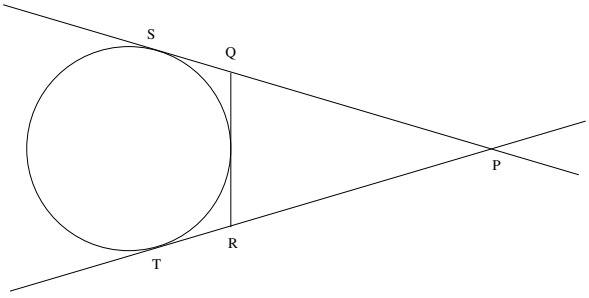
11. Consider a trapezoid one of whose bases is twice the length of the other. Let the length of the shorter base be a. In addition the angles at the longer base are $45°$. What is the area of the quadrilateral whose vertices are the midpoints of each of the sides of the trapezoid?

(A) $\frac{3a^2}{4}$ (B) $\frac{a^2}{4}$ (C) $2a^2$ (D) $\sqrt{2}a^2$ (E) None of the answers (A) through (D) is correct.

12. Find the area bounded by the x-axis on the bottom, the circle $(x-4)^2+y^2=4$ on the left, and the line $y=4-x$ on the right.

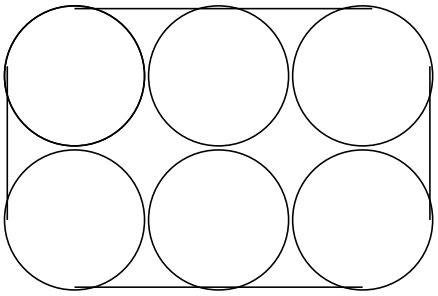
(A) $4\pi-2$ (B) $\pi-2$ (C) $\frac{\pi}{2}$ (D) 2π (E) None of the answers (A) through (D) is correct.

13. In the figure, $PS = 5$ ft. What is the perimeter of triangle PQR?



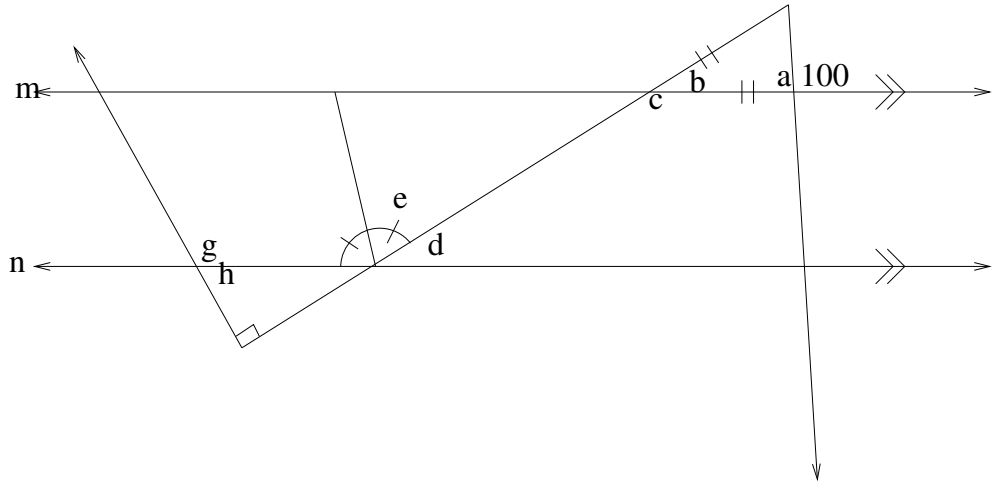
(A) 10 ft. (B) $5\sqrt{2}$ ft. (C) 7.5 ft (D) 14 ft.
(E) None of the answers (A) through (D) is correct.

14. Six cylindrical barrels are tied together with a strap as shown in the figure. If each barrel has a diameter of 6 feet, how long must the strap be?



(A) 60 ft. (B) $6\pi + 36$ ft. (C) $3\pi + 36$ ft. (D) $12\pi + 12$ ft. (E) 30 ft.

15. In the figure, the lines m and n are parallel. What is the average of the measures of angles e and g?

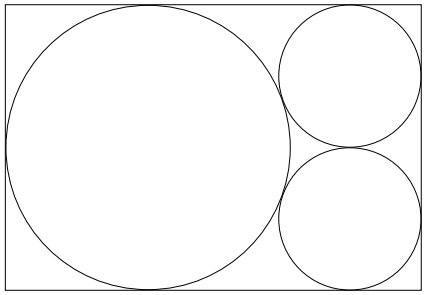


(A) 85° (B) 87.5° (C) 90° (D) 95° (E) 97.5°

16. A semicircle with area of $x\pi$ is marked by seven points equally spaced along the half arc of the semicircle such that two of the seven points form the endpoints of the diameter. Find the greatest area formed by any two of those seven points and the center of the circle.

(A) x (B) $2x$ (C) $\sqrt{\frac{3}{2}}x$ (D) $\frac{1}{2}x$ (E) $\sqrt{\frac{2}{3}}x$

17. A rectangle contains three circles as in the diagram. If the height of the rectangle is 2, then the width of the rectangle is



(A) $\frac{3}{2}+\sqrt{2}$ (B) 3 (C) $2\sqrt{2}$ (D) $\frac{3+\sqrt{3}}{2}$ (E) None of the answers (A) through (D) is correct.

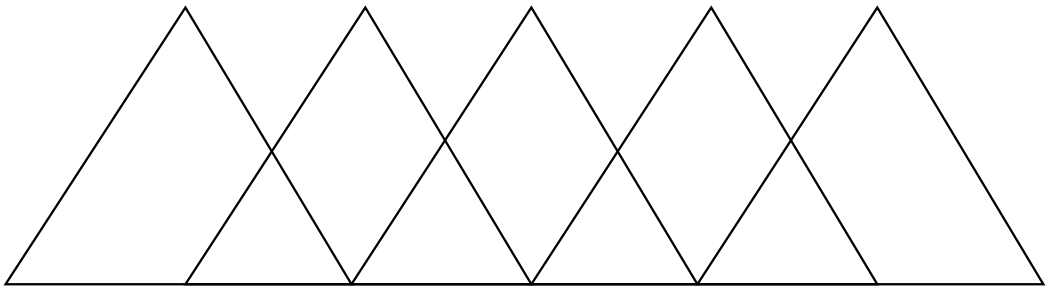
18. In the rectangle $ABCD$ the side $AD = 1$, P is on the side $\overline{AB}$, and $\overline{DB}$ and $\overline{DP}$ trisect $\angle ADC$. What is the perimeter of $\triangle BDP$?

(A) $3+\frac{\sqrt{3}}{3}$ (B) $2+\frac{4\sqrt{3}}{3}$ (C) $2+2\sqrt{2}$ (D) $\frac{3+3\sqrt{5}}{2}$ (E) $2+\frac{5\sqrt{3}}{3}$

19. Medians $\overline{BD}$ and $\overline{CE}$ of $\triangle ABC$ are perpendicular, $BD = 8$ and $CE = 12$. Find the area of $\triangle ABC$.

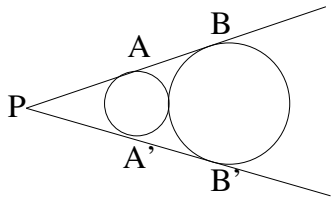
(A) 24 (B) 32 (C) 48 (D) 64 (E) 96

20. Five equilateral triangles, each with side length $2\sqrt{3}$, are arranged so they are all on the same side of a line containing one side of each. Along this line, the midpoint of the base of one triangle is the vertex of the next, as shown. What is the area of the region of the plane that is covered by the union of the triangular regions?



(A) 10 (B) 12 (C) 15 (D) $10\sqrt{3}$ (E) $12\sqrt{3}$

21. Two circles are externally tangent. Lines PAB and $PA'B'$ are common tangents with A and A' on the smaller circle and B and B' on the larger circle, and $PA = AB = 4$. What is the area of the smaller circle?



(A) 1.44π (B) 2π (C) 2.56π (D) $\sqrt{8}\pi$ (E) 4π

22. Let $ABCD$ be a trapezoid with bases $\overline{AB}$ and $\overline{CD}$. If $AB = 52, BC = 12$, $CD = 39$, and $DA = 5$, find the area of $ABCD$.

(A) 182 (B) 195 (C) 210 (D) 234 (E) 260

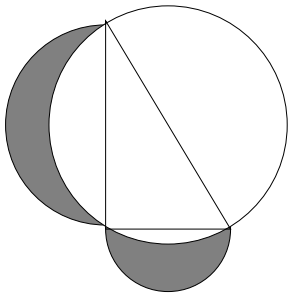
23. Let $ABCD$ be a trapezoid with bases $\overline{AB}$ and $\overline{CD}$ such that $AB = 2CD$. Let E be the point of intersection of the diagonals AC and BD. If $AC = 11$, find the length of the segment EC.

(A) $3\frac{2}{3}$ (B) $3\frac{3}{4}$ (C) 4 (D) $3\frac{1}{2}$ (E) 3

24. A pony is tethered to the corner of a rectangular barn. If the length of the rope is 5, and the barn has length 4 and width 3, what is the maximum area that is accessible to the pony? (The pony cannot go inside the barn.)

(A) 12π (B) 15π (C) 16π (D) 18π (E) 20π

25. Circles are constructed on each of the three sides of a right triangle ABC using the sides as diameters. If the area of the triangle is 12 square units, what is the total area of the two smaller circles that lies outside the largest circle (shaded in the figure)?



(A) 3π square unites (B) 9 square unites (C) 12 square unites (D) 4π square unites
(E) 5π square unites