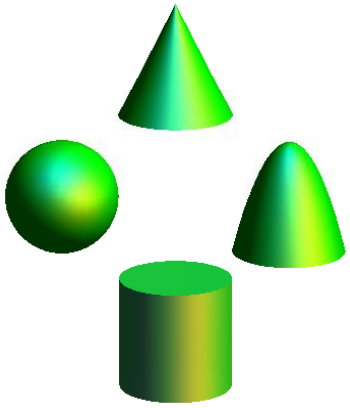
2011
High School Math Contest

Geometry Exam



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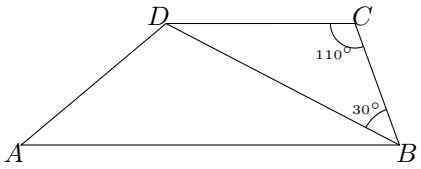
Do's and Don'ts:

- **Do NOT** open this booklet until you are instructed to do so.
- **NO Calculators (Or other electronic devices)**
- Contestants with electronic devices (on or off) will be **disqualified!**
  And their **team** will be **disqualified!**
  (The other team members will continue to participate in the individual contest.)

DIRECTIONS:

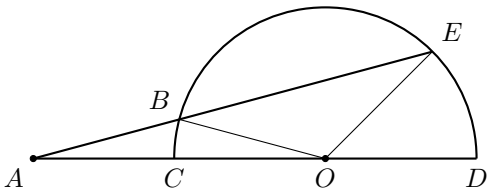
- **The Score Card:**
  - For the Geometry exam, you must use the scorecard highlighted in **GREEN**.
  - Write:
    - Your **NAME** on the "**name line**" (of course).
    - "**GEOMETRY**" on the "**subject line**".
    - Your **SCHOOL** on the "**Date Line**"
- **Clearly mark** ONE bubble using **#2 PENCIL**.
  - Light marks will be counted as unmarked!
  - **Completely erase** any changes.
- You **can** write on this test booklet. (But the test booklet will not be graded.)
- **Tie Breakers**: In case of ties, the person with the least number of *wrong answers* wins.
  (Thus, a ***blank*** is better than ***incorrect!***)
- **The Exam:** **30** problems, **70** minutes.
- **WAIT** for the signal to begin.

1. In the trapezoid $ABCD$, sides AB and CD are parallel and diagonal BD and side AD have equal length. If $\angle DCB = 110^\circ$ and $\angle CBD = 30^\circ$, find $\angle ADB$.



(a) 30° (b) 150° (c) 100° (d) 120° (e) None of the answers (a) through (d) is correct.

2. Let CD be the diameter of a semi-circle with center O. Let point A lie on the extension of DC past C; point E lies on the semi-circle, and B be the point of intersection (distinct from E) of the line segment AE with the semi-circle. If $\overline{AB} = \overline{OD}$ and $\angle EOD = 45^\circ$, find $\angle BAO$.

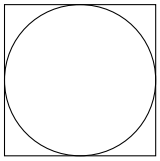


(a) 15° (b) 45° (c) 30° (d) 22.5° (e) None of the answers (a) through (d) is correct.

3. Rocky rock climber is sitting under a tree 200 ft from the base of a cliff. The angle of elevation to the top of the cliff where his big red dog Clifford is barking at a squirrel is 30 degrees. Approximately how tall is the cliff?

(a) $200\frac{\sqrt{3}}{3}$ ft (b) $200\frac{\sqrt{3}}{2}$ ft (c) $200\sqrt{3}$ ft (d) $\frac{\sqrt{3}}{600}$ ft
(e) None of the answers (a) through (d) is correct.

4. The figure to the right shows a circle inscribed into a square. A dart player throws at random onto the square board. What is the probability that the dart will hit outside of the circle?



(a) $\frac{\pi}{4}$ (b) $\frac{1}{4}$ (c) $1 - \frac{\pi}{4}$ (d) $1 - \pi r^2$ (e) None of the answers (a) through (d) is correct.

5. After a dilation, (-5, -2) is the image of (-20, -8). What are the coordinates of the image of (-40, -4) after the same dilation?

(a) $(-10, -1)$ (b) $(-\frac{5}{2}, -1)$ (c) $(-25, 2)$ (d) $(-160, -16)$
(e) None of the answers (a) through (d) is correct.

6. An isosceles right triangle has a hypotenuse of length 4 units. What is the area of the triangle?

(a) 2 square units (b) 4 square units (c) 6 square units (d) 8 square units
(e) 16 square units

7. Let ABC be a right triangle with $\angle BAC = 30^\circ$. Let BD be the altitude from B to AC and E be the midpoint between A and C. Find the length of the segment DE if $\overline{AB} = 3\sqrt{3}$ cm.

(a) 1 cm (b) 2 cm (c) $\frac{3\sqrt{3}}{2}$ cm (d) $\frac{3}{2}$ cm (e) 3 cm

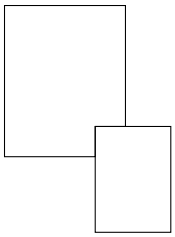
8. An equilateral triangle and a regular hexagon have equal perimeters. If the area of the triangle is 2 square units, then the area of the hexagon is

(a) 2 sq. units (b) 3 sq. units (c) 4 sq. units (d) 6 sq. units (e) 9 sq. units

9. Triangles ABC and DEF are similar. The length of the sides of ABC are $8+3x$, $2x+96$, and $92+2x$. The perimeter of ABC is 476. The length of the longest side of DEF is $24+6x$, what is the perimeter of DEF?

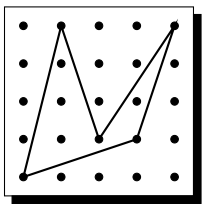
   (a) 264 (b) 714 (c) 317 (d) 952 (e) None of the answers (a) through (d) is correct.

10. A rectangle of length 10 centimeters and width 8 centimeters overlaps with another rectangle of length 7 centimeters and width 5 centimeters as shown below. What is the absolute value of the difference between the area of the two non-overlapping regions of the rectangles?



   (a) 80 sq units (b) 35 sq units (c) 45 sq units (d) 55 sq units
   (e) None of the answers (a) through (d) is correct.

11. What is the area of the enclosed shape assuming that the area of the entire board is 16 square units?



   (a) 4 sq. units (b) $4\frac{1}{2}$ sq. units (c) 5 sq. units (d) $5\frac{1}{2}$ sq. units
   (e) None of the answers (a) through (d) is correct.

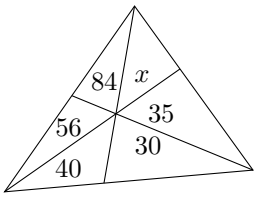
12. In the triangle ABC, D is the midpoint of AB, E is the midpoint of DB, and F is the midpoint of BC. If the area of $\triangle ABC$ is 96 sq. units, find the area of $\triangle AEF$.

   (a) 48 sq. units (b) 36 sq. units (c) 24 sq. units (d) 32 sq. units
   (e) None of the answers (a) through (d) is correct.

13. A ball of clay 10 cm in diameter is rolled into a cylinder 10 cm long. Find the radius of the resulting cylinder.

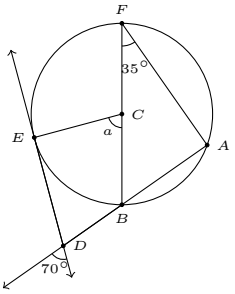
   (a) $\frac{5\sqrt{6}}{3}$ cm (b) $\frac{5\sqrt{2}}{\sqrt{3}}$ in (c) 5 cm (d) $\frac{5\sqrt{3}}{\sqrt{2}}$ cm
   (e) None of the answers (a) through (d) is correct.

14. As shown in the figure, $\triangle ABC$ is divided into six smaller triangles by lines drown from the vertices through a common interior point. The areas of the five of these triangles are as indicated. Find the area of $\triangle ABC$.



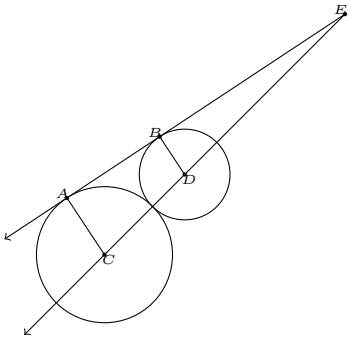
   (a) 285 (b) 315 (c) 318 (d) 300 (e) None of the answers (a) through (d) is correct.

15. In the figure, C is the center of the circle and line $\overleftrightarrow{ED}$ is tangent to circle C at point E. What is the measure of angle $\angle a$?



(a) 65° (b) 70° (c) 75° (d) 80° (e) 85°

16. In the figure to the right C and D are the centers of the circles and A and B are points of tangency. If $\overline{AC}$ has length 3 units and $\overline{BD}$ has length 2 units, then what is the length of $\overline{CE}$?



(a) 10 units (b) 15 units (c) 16 units (d) 18 units (e) 20 units

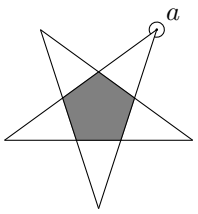
17. A box has a depth of 2 ft, a height of 3 ft, and a width of 5 ft. What is the length of its longest diagonal?

(a) 5 ft (b) $\sqrt{29}$ ft (c) $\sqrt{34}$ ft (d) $\sqrt{38}$ ft (e) $\sqrt{40}$ ft

18. A cube is inscribed into a sphere. If the sphere has a volume of $\frac{500}{3}\pi$ cm^3, then what is the volume of the cube?

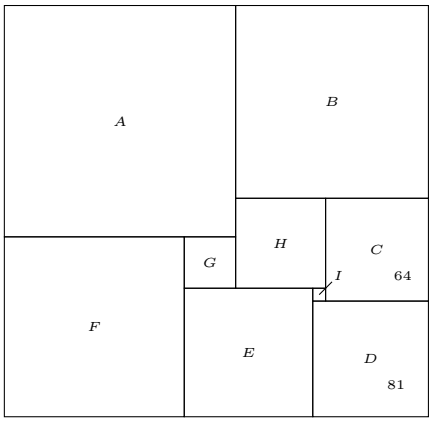
(a) 1000 cm^3 (b) $\frac{500}{9}$ cm^3 (c) $\frac{1000\sqrt{2}}{3}\pi$ cm^3 (d) $\frac{1000\sqrt{3}}{9}$ cm^3
(e) None of the answers (a) through (d) is correct.

19. To the right is a star. The shaded area is a regular pentagon. Find the measure of the reflex angle a.



(a) 36° (b) 280° (c) 300° (d) 308° (e) 324°

20. The figure to the right shows a rectangle that has been divided into 9 smaller squares, each having different areas. Suppose that the areas of the square C and D are 64 and 81 respectively. What is the area of square F?

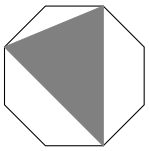


(a) 144 sq. units (b) 169 sq. units (c) 196 sq. units (d) 255 sq. units
(e) None of the answers (a) through (d) is correct.

21. If three balls are packed in a cylindrical can whose diameter equals that of a ball and whose height is 3 times the diameter, which estimate best represents the fraction of space that is unused?

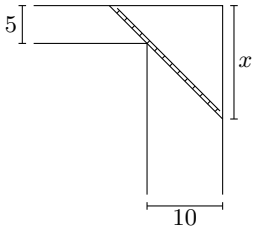
(a) $\frac{1}{6}$ (b) $\frac{1}{5}$ (c) $\frac{1}{4}$ (d) $\frac{1}{3}$ (e) $\frac{1}{2}$

22. A regular octagon has side length 2 cm. What is the area of the shaded region?



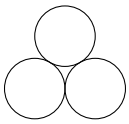
(a) $3 + 2\sqrt{2}$ sq. cm (b) $5/2 + (3/2)\sqrt{3}$ sq. cm (c) $4 + 3\sqrt{2}$ sq. cm
(d) $1/2 + (3/4)\sqrt{2}$ sq. cm (e) 3 sq. cm

23. A ladder is gotten stuck in a hallway with dimensions given to the right. If $x = 15$ ft how long is the ladder?



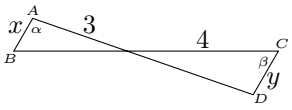
(a) $30\sqrt{2}$ (b) $(3/2)\sqrt{125}$ (c) $\sqrt{125}$ (d) $15\sqrt{2}$ (e) 20

24. Three logs of diameter 8 cm are stacked in a pile in the following manner. How tall is the pile?



(a) $8 + 4\sqrt{3}$ cm (b) $4\sqrt{3}$ cm (c) 16 cm (d) $8\sqrt{3}$ cm (e) $8 + 4\sqrt{2}$ cm

25. Assume AB is parallel to CD. The length of AB is expressed as x, and the length of CD is expressed as y. Which of the following is an expression for the length of y?

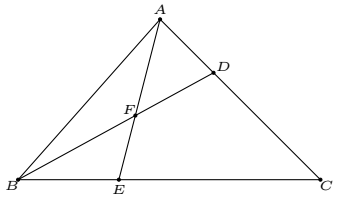


(a) $\sqrt{7 + x^2}$ (b) $x\sqrt{25 - 24\cos(180 - \beta - \alpha)}$ (c) $\frac{4}{3}x$ (d) $\frac{4\sin(\beta)}{3\sin(\alpha)}x$ (e) $\frac{3\cos(\beta)}{4\cos(\alpha)}x$

26. A circle of radius r is inscribed in a triangle. If the perimeter of the triangle is P and the area is A, find $\frac{P}{A}$.

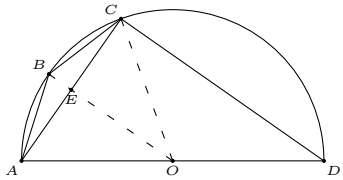
(a) $\frac{2}{r}$ (b) $\frac{r}{2}$ (c) $\frac{2}{r+1}$ (d) $\frac{3}{r}$ (e) None of the answers (a) through (d) is correct.

27. In triangle ABC, point D divides side AC in the ratio $1:2$. Let E be the point of intersection of BC and AF, where F is the midpoint of BD. Find the ratio in which E divides BC.



(a) 1:2 (b) 3:2 (c) 1:3 (d) 3:1 (e) None of the answers (a) through (d) is correct.

28. Quadrilateral $ABCD$ is inscribed in a circle with side AD as a diameter of length 4 cm. If sides AB and BC each have length 1 cm, find the length of the side CD.

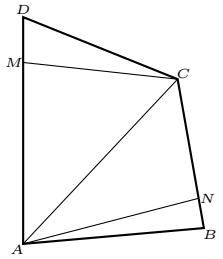


(a) $\frac{7}{2}$ cm (b) 2 cm (c) 1 cm (d) $\frac{7}{4}$ cm (e) None of the answers (a) through (d) is correct.

29. The difference in the areas of two similar triangles is 18 cm^2 and the ratio of the larger area to the smaller area is the square of an integer. The area of the smaller triangle in cm^2, is an integer and one of its sides is 3 cm. Find the corresponding side of the larger triangle.

(a) 4 cm (b) 6 cm (c) 3 cm (d) 9 cm (e) None of the answers (a) through (d) is correct.

30. Let $ABCD$ be a quadrilateral, and let M and N be points on the sides AD and BC respectively such that $\overline{MD} = \frac{1}{k}\overline{AD}$ and $\overline{NB} = \frac{1}{k}\overline{BC}$, where k is non-zero positive number. Find the ratio of the areas of $AMCN$ and $ABCD$.



(a) $(k-1):k^2$ (b) $(k-1):k$ (c) $k:(k-1)$ (d) $1:k$
(e) None of the answers (a) through (d) is correct.