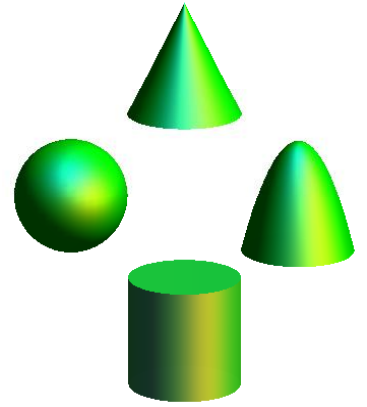




2012
High School Math Contest

Geometry
Exam



Lenoir-Rhyne University
*Donald and Helen Schort School of
Mathematics and Computing Sciences*

This exam has been prepared by the following faculty from **Western Carolina University**:

Risto Atanasov

Mark Budden

Nathan Borchelt

Andrew Chockla

Geoff Goehle

Axelle Faughn

John Wagaman

Do's and Don'ts:

- **Do NOT** open this booklet until you are instructed to do so.
- **NO Calculators** (Or other electronic devices)
- Contestants with electronic devices (on or off) will be **disqualified!**
And their **team** will be **disqualified!**
(The other team members will continue to participate in the individual contest.)

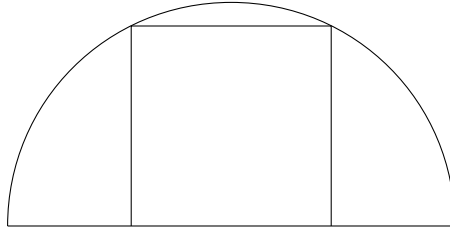
DIRECTIONS:

- **The Score Card:**
 - For the ALGEBRA II exam, you must use the scorecard highlighted in **GREEN**.
 - Write:
 - Your **NAME** on the “**name line**” (of course).
 - “**GEOMETRY**” on the “**subject line**”.
 - Your **SCHOOL** on the “**Date Line**”
- **Clearly mark** ONE bubble using **#2 PENCIL**.
 - Light marks will be counted as unmarked!
 - **Completely erase** any changes.
- You **can** write on this test booklet. (But the test booklet will not be graded.)
- **Tie Breakers:** In case of ties, the person with the least number of *wrong answers* wins.
(A *blank* is better than *incorrect!*)
- **The Exam:** **30** problems, **70** minutes.
- **WAIT** for the signal to begin.

1. Circles of diameter 1 inch and 3 inches have the same center. The smaller circle is painted gold, and the portion outside the smaller circle and inside the larger circle is painted purple. What is the ratio of the purple-painted area to the gold-painted area?

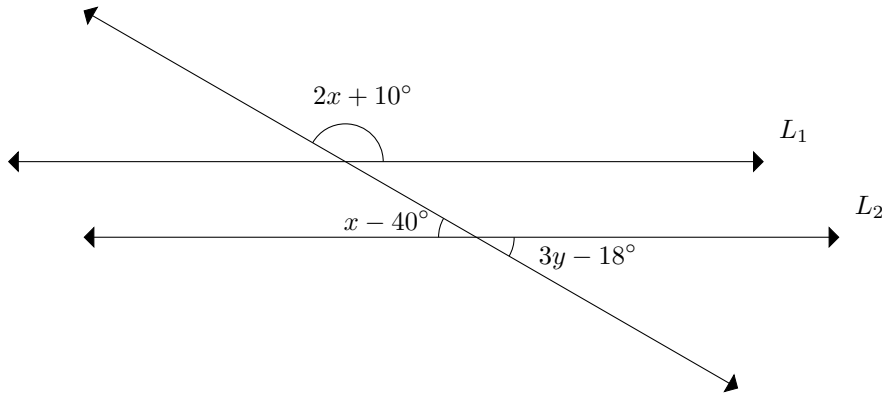
(A) 4:1 (B) 6:1 (C) 8:1 (D) 9:1 (E) 10:1

2. A square of area 28 is inscribed in a semicircle as shown. What is the area of the semicircle?



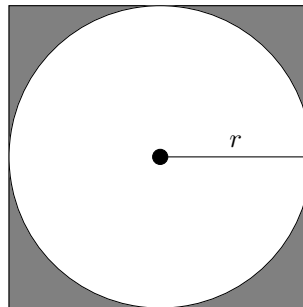
(A) $\frac{25\pi}{2}$ (B) $\frac{35\pi}{2}$ (C) $\frac{37\pi}{3}$ (D) $\frac{41\pi}{3}$ (E) $\frac{39\pi}{4}$

3. Given that lines L_1 and L_2 are parallel, find the value of $x + y$.



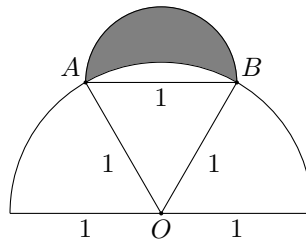
(A) 74° (B) 92° (C) 63° (D) 86° (E) 79°

4. Determine the area of the shaded portion in the following diagram.



(A) $\pi r^2 - 4$ (B) $4\pi r^2 - 2r$ (C) $4r^2 - \pi r^2$ (D) $4r^2 - 4\pi$ (E) $8\pi r^2$

5. The length of one side of an equiangular triangle is 3 cm. What is the area of the triangle?
 (A) $\frac{9\sqrt{3}}{4}$ cm² (B) $3\sqrt{3}$ cm² (C) $\frac{9\sqrt{3}}{2}$ cm²
 (D) $9\sqrt{3}$ cm² (E) None of the answers (A) through (D) is correct.
6. A triangle has side lengths 10, 10, and 12. A rectangle has width 4 and area equal to the area of the triangle. What is the perimeter of this rectangle?
 (A) 26 (B) 32 (C) 22 (D) 30 (E) 28
7. Two circles, of respective radii 4 and 1, touch each other externally. A common tangent touches one of them at the point P and the other at Q . Then PQ has length
 (A) 1 (B) 4 (C) 15 (D) 25 (E) None of the answers (A) through (D) is correct.
8. Determine the slant height of a pyramid with a square base that has four congruent triangles as faces with surface area of 832 m² and base side length of 16 m.
 (A) 20 m (B) 13 m (C) 16 m (D) 18 m (E) 21 m
9. A community garden is being constructed in the shape of a trapezoid with bases of 40 feet and 60 feet. The distance between the bases is 25 feet. It is determined that 1 bag of organic topsoil is needed to cover 15 square feet of the garden. What is the minimum number of bags of topsoil needed for this project?
 (A) 125 (B) 84 (C) 92 (D) 76 (E) 112
10. The sides of a triangle have lengths of 15, 20, and 25. What is the length of the shortest altitude?
 (A) 6 (B) 12 (C) 13 (D) 15 (E) None of the answers (A) through (D) is correct.
11. A semicircle of diameter 1 sits on top of a semicircle of diameter 2, as shown. The shaded area inside the smaller semicircle and outside the larger semicircle is called a lune. What is the area of this lune?

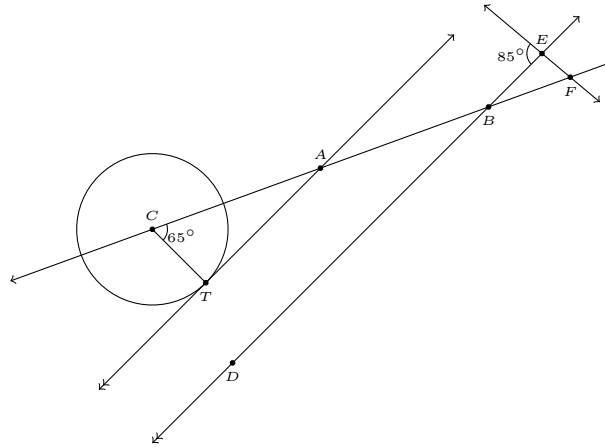


- (A) $\frac{1}{12}(2\pi - 3\sqrt{3})$ (B) $\frac{1}{12}(3\sqrt{3} - \pi)$ (C) $\frac{1}{24}(6\sqrt{3} - \pi)$
 (D) $\frac{1}{12}(3\sqrt{3} + \pi)$ (E) None of the answers (A) through (D) is correct.
12. Let \overline{AB} and \overline{CD} be perpendicular diameters of a circle with center O . Chord \overline{DF} intersects \overline{AB} at E with $DE = 6$ and $EF = 2$. What is the area of the circle?
 (A) 23π (B) $\frac{47}{2}\pi$ (C) 24π
 (D) 25π (E) None of the answers (A) through (D) is correct.

13. The length of one side of a regular octagon is 2 ft. What is the area of the octagon?

- (A) 8 ft^2 (B) $8\sqrt{2} \text{ ft}^2$ (C) $8(1 + \sqrt{2}) \text{ ft}^2$
 (D) 28 ft^2 (E) None of the answers (A) through (D) is correct.

14. In the picture below, \overleftrightarrow{AT} is tangent to the circle centered at C and \overleftrightarrow{AT} is parallel to \overleftrightarrow{BD} . What is the measure of $\angle BFE$?



- (A) 50° (B) 60° (C) 65°
 (D) 70° (E) None of the answers (A) through (D) is correct.

15. A circle of radius 2 cm is inscribed in an equilateral triangle, which is inscribed in another circle. What is the diameter of the larger circle?

- (A) 4 cm (B) 8 cm (C) $4\sqrt{3}$ cm
 (D) 16 cm (E) None of the answers (A) through (D) is correct.

16. Fred leaves his house and walks 5 miles to the west. He then heads north for 2 miles before heading northeast for 3 miles. Finally, he goes another 2 miles southeast. How far is Fred from his house?

- (A) $\sqrt{42 - 23\sqrt{2}}$ miles (B) $\sqrt{114}$ miles (C) 3 miles
 (D) $\sqrt{114 - 41\sqrt{2}}$ miles (E) None of the answers (A) through (D) is correct.

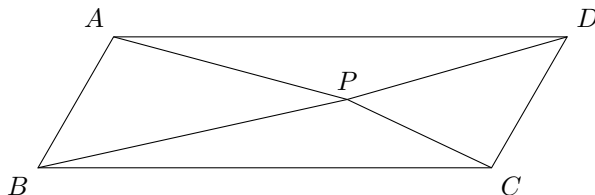
17. A right circular cone has a height of 12 cm and a radius of 5 cm. What is its lateral surface area?

- (A) 30 cm^2 (B) $\frac{65\pi}{3} \text{ cm}^2$ (C) $300\pi \text{ cm}^2$
 (D) $65\pi \text{ cm}^2$ (E) None of the answers (A) through (D) is correct.

18. You have 5 sticks, of lengths 1, 2, 3, 4 and 5 feet. The number of non-congruent triangles that can be formed by choosing three of the sticks to make the sides is

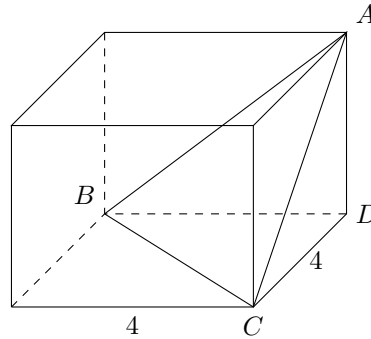
- (A) 3 (B) 5 (C) 8 (D) 9 (E) None of the answers (A) through (D) is correct.

19. P is a point in a parallelogram $ABCD$. If the area of the triangle PAD is one-third that of $ABCD$, and the area of triangle PCB is 6 cm^2 , then the area of the parallelogram is



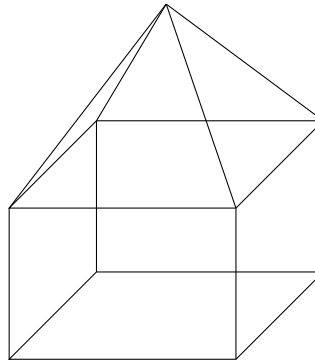
- (A) 24 (B) 36 (C) 48
 (D) 60 (E) None of the answers (A) through (D) is correct.
20. A glass box 7 in by 12 in by 18 in, closed on all sides, is partly filled with colored water. When the box is placed on one of its 7 by 12 sides, the water level is 15 in above the table. When the box is placed on one of its 7 by 18 sides the water level above the table, in inches, will be
- (A) 7.5 in (B) 9 in (C) 10 in
 (D) 12.5 in (E) None of the answers (A) through (D) is correct.
21. In the quadrilateral $ABCD$, $AB = \sqrt{6}$, $m(\angle ABC) = 135^\circ$, $BC = 5 - \sqrt{3}$, $m(\angle BCD) = 120^\circ$, and $CD = 6$. Then the length of AD is
- (A) 8 (B) $\sqrt{73}$ (C) $2\sqrt{19}$
 (D) $\sqrt{91}$ (E) None of the answers (A) through (D) is correct.
22. A right triangle with integer side lengths a , b , and c satisfies $a < b < c$ and $c - a = 9$. What is the area of the right triangle?
- (A) 72 (B) 84 (C) 30 (D) 76 (E) 60
23. In rectangle $ABCD$, we have $AB = 5$ and $BC = 3$. Points F and G are on \overline{CD} with $DF = 1$ and $CG = 2$, and lines AF and BG intersect at E . What is the area of $\triangle AEB$?
- (A) 10 (B) $\frac{21}{2}$ (C) 12
 (D) $\frac{25}{2}$ (E) None of the answers (A) through (D) is correct.
24. An acute isosceles triangle BAC is inscribed in a circle. Tangents to the circle are drawn at B and C , meeting at a point D with $\angle ABC = m(\angle ACB) = 2(\angle CDB)$. What is the radian measure of $\angle BAC$?
- (A) $\frac{3\pi}{7}$ (B) $\frac{4\pi}{9}$ (C) $\frac{6\pi}{13}$ (D) $\frac{7\pi}{15}$ (E) None of the answers (A) through (D) is correct.

25. On a $4 \times 4 \times 3$ rectangular parallelepiped, vertices A , B , and C are adjacent to vertex D (as shown on the figure). Consider the plane containing the points A , B , and C . What is the perpendicular distance from D to this plane?



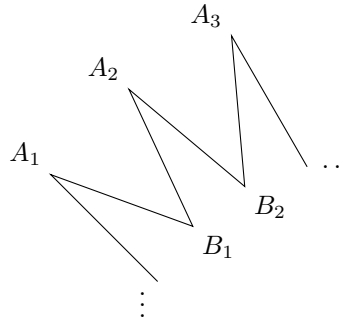
- (A) $\frac{6\sqrt{34}}{17}$ (B) $\sqrt{5}$ (C) $\sqrt{6}$
 (D) $2\sqrt{2}$ (E) None of the answers (A) through (D) is correct.

26. The base of the gazebo shown below is 3 meters square and the walls are 2 meters high. The roof-bracers from the top of the roof to the corners on top of the walls are 3 meters. The height, in meters, of the top of the roof from the ground is

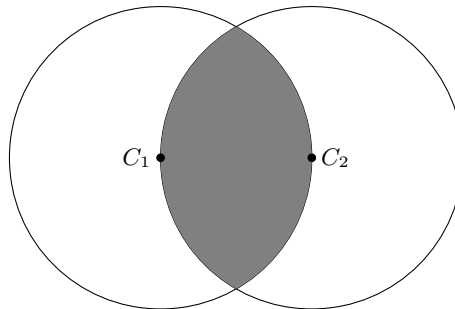


- (A) $2 + \frac{\sqrt{2}}{2}$ m (B) $\sqrt{\frac{17}{2}}$ m (C) $2 + \frac{3\sqrt{2}}{2}$ m
 (D) $\sqrt{22}$ m (E) None of the answers (A) through (D) is correct.

27. A typical “ n -pointed regular star” is shown. It is a polygon all of whose $2n$ edges have the same length, the acute angles A_1, A_2, \dots, A_n are all congruent, and the acute angles B_1, B_2, \dots, B_n are all congruent. Suppose $m(\angle B_1) - m(\angle A_1) = 10^\circ$. What is the value of n ?



- (A) 36 (B) 18 (C) 24
 (D) 60 (E) None of the answers (A) through (D) is correct.
28. The centers of the circle C_1 and C_2 in the pictures below are 4 cm apart. What is the area of the shaded region?



- (A) $8\sqrt{3}$ cm² (B) $\frac{16\pi}{3}$ cm² (C) $\frac{16\pi}{3} - 4\sqrt{3}$ cm²
 (D) $\frac{32\pi}{3} - 8\sqrt{3}$ cm² (E) None of the answers (A) through (D) is correct.
29. A 33×75 rectangle is divided into unit squares. One diagonal of the rectangle is drawn. The number of unit squares containing a positive length of this diagonal is
- (A) 100 (B) 101 (C) 102
 (D) 103 (E) None of the answers (A) through (D) is correct.
30. If n metal spheres 3 in in diameter are to be melted down to form one metal sphere of diameter 10 in, then the minimum value of n satisfies
- (A) $n < 5$ (B) $5 \leq n < 10$ (C) $10 \leq n < 15$ (D) $15 \leq n < 20$ (E) $n \geq 20$