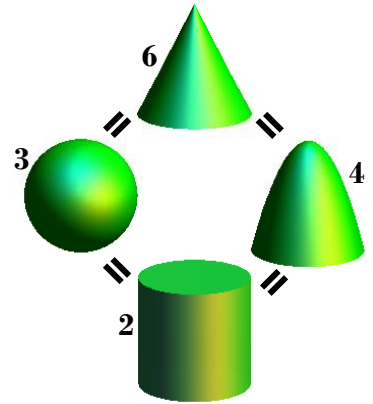




2012
High School Math Contest

Algebra II
Exam

Lenoir-Rhyne University
*Donald and Helen Schort School of
Mathematics and Computing Sciences*



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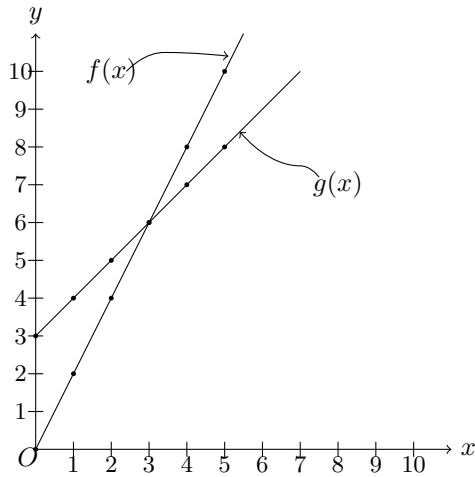
Do's and Don'ts:

- **Do NOT** open this booklet until you are instructed to do so.
- **NO Calculators** (Or other electronic devices)
- Contestants with electronic devices (on or off) will be **disqualified!**
And their **team** will be **disqualified!**
(The other team members will continue to participate in the individual contest.)

DIRECTIONS:

- **The Score Card:**
 - For the ALGEBRA II exam, you must use the scorecard highlighted in **ORANGE**.
 - Write:
 - Your **NAME** on the “**name line**” (of course).
 - “**ALGEBRA II**” on the “**subject line**”.
 - Your **SCHOOL** on the “**Date Line**”
- **Clearly mark** ONE bubble using **#2 PENCIL**.
 - Light marks will be counted as unmarked!
 - **Completely erase** any changes.
- You **can** write on this test booklet. (But the test booklet will not be graded.)
- **Tie Breakers:** In case of ties, the person with the least number of *wrong answers* wins.
(A *blank* is better than *incorrect!*)
- **The Exam:** **30** problems, **70** minutes.
- **WAIT** for the signal to begin.

1. Refer to the graph shown below.



What is the value of $f(g(2))$?

- (A) 4 (B) 5 (C) 8 (D) 7 (E) 10

2. Assuming that a solution exists, what are the dimensions of M ?

$$\begin{bmatrix} 1 & 2 \\ 3 & 4 \\ 5 & 6 \end{bmatrix} M = \begin{bmatrix} 2 & 3 \\ 5 & -10 \\ 0 & 1 \end{bmatrix}$$

- (A) (2, 2) (B) (2, 3) (C) (3, 2)
 (D) (3, 3) (E) None of the answers (A) through (D) is correct.

3. What is the logarithmic form of the equation $y = 2^{1/4}$?

- (A) $\log_y 2 = \frac{1}{4}$ (B) $\log_2 y = \frac{1}{4}$ (C) $\log_{\frac{1}{4}} 2 = y$ (D) $\log_{\frac{1}{4}} y = 2$ (E) $\log_2 \left(\frac{1}{4}\right) = y$

4. Consider the equation $e^{x^{2x+1}} = 1$ and the following three statements.

- (1) $x = 0$ is the solution of the equation.
 (2) $x = \sqrt{\frac{\ln 1}{2}}$ is the solution of the equation.
 (3) $x = \frac{\sqrt{\ln 1}}{2}$ is the solution of the equation.

Which of the following is true?

- (A) Only statement (1) is true.
 (B) Only statement (2) is true.
 (C) Only statement (3) is true.
 (D) Exactly two of the statements (1), (2), (3) are true.
 (E) All three statements (1), (2), and (3) are true.

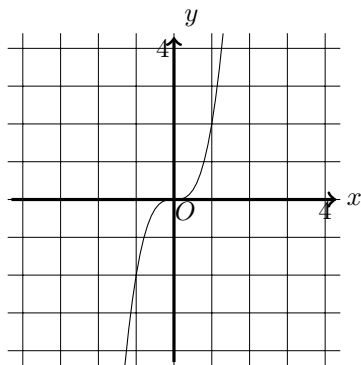
5. Simplify $\frac{1-x}{1-\frac{1}{1-x}}$

- (A) 1 (B) $1-x$ (C) $1+x$ (D) $x-1$ (E) $(1-x)^2$

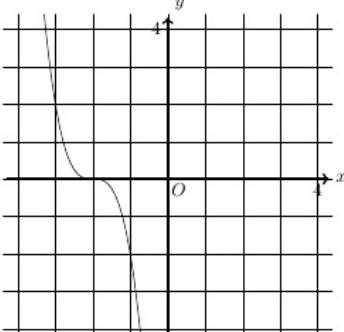
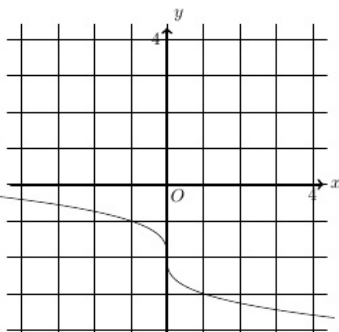
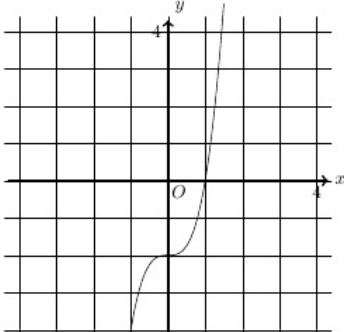
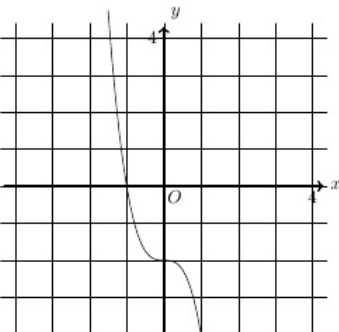
6. Suppose x varies jointly as 2^{-y} and z^2 , and $x = 1$ when $y = 1$ and $z = 2$. What is the value of x when $y = -3$ and $z = \frac{1}{2}$?

- (A) $\frac{1}{2}$ (B) 1 (C) 2 (D) 4 (E) 8

7.



The graph of $f(x) = 2x^3$ is shown above. Which of the following is the graph of $-f(x) - 2$?

- (A) 
- (B) 
- (C) 
- (D) 

(E) None of the answers (A) through (D) is correct.

8. If $f(x) = \frac{2x+1}{-3}$ and $g(x) = 2x^2 + 2$, then $f(g(2)) =$
 (A) -3 (B) -5 (C) -7 (D) $-7\frac{5}{9}$ (E) $-16\frac{2}{3}$
9. Solve for x : $\log_x\left(\frac{1}{9}\right) = \frac{-2}{3}$
 (A) $x = 27$ (B) $x = 3$ (C) $x = \frac{2}{9}$ (D) $x = 9$ (E) $x = \frac{1}{3}$
10. Which of the following polynomials is the result of distributing the following: $(x - (3 + i))(x - (3 - i))$
 (A) $x^2 - 6x + 2xi - 6i + 8$ (B) $x^2 - 6x - 10$ (C) $x^2 - 9x + 8$
 (D) $x^2 + 6x - 2xi + 6i + 8$ (E) $x^2 - 6x + 10$
11. Which of the following is equivalent to $\frac{1+5i}{1-10i}$?
 (A) $\frac{15i-49}{11}$ (B) $\frac{15i+49}{11}$ (C) $\frac{15i+51}{11}$ (D) $\frac{15i-49}{101}$ (E) $\frac{15i+51}{101}$
12. In the xy -plane, a horizontal line intersects the y -axis at the point $(0, 4)$. This line also intersects a circle in two points. The circle has a radius of 5 and its center is at the origin. What are the coordinates of the two points of intersection?
 (A) $(1, 2)$ and $(2, 1)$ (B) $(2, 1)$ and $(2, -1)$ (C) $(3, 4)$ and $(3, -4)$
 (D) $(3, 4)$ and $(-3, 4)$ (E) $(5, 0)$ and $(-5, 0)$
13. What geometric shape do the vertices of the following system make up?

$$\begin{aligned} 6x - 3y &\geq -2 \\ 2y &\geq 4 \\ 2x - y &\leq 2 \\ x &\leq 7 \end{aligned}$$

- (A) Triangle (B) Rectangle (C) Parallelogram (D) Trapezoid (E) Rhombus
14. Find the inverse function of $y = \frac{\sqrt{6x+8}}{4}$
 (A) $y = \frac{8x^2-8}{3}$ (B) $y = \frac{8x^2-4}{3}$ (C) $y = \frac{8x^2+4}{3}$ (D) $y = \frac{16x^2-4}{3}$ (E) $y = \frac{8x^2+8}{3}$
15. Consider the equation $3^{6x} = 30$ and the following three statements.

- (1) $x = \frac{\ln 30}{6 \ln 3}$ is the solution of the equation.
 (2) $x = \frac{1}{6} \left(1 + \frac{\ln 10}{\ln 3}\right)$ is the solution of the equation.
 (3) $x = \frac{1+\ln 7}{6}$ is the solution of the equation.

Which of the following is true?

- (A) Only statement (1) is true.
 (B) Only statement (2) is true.
 (C) Only statement (3) is true.
 (D) Exactly two of the statements (1), (2), (3) are true.
 (E) All three statements (1), (2), and (3) are true.

16. Divide $(2x^3 - 7x^2 - 2x - 15)$ by $(x - 4)$.
- (A) $2x^2 + x + 2 + \frac{7}{x-4}$ (B) $2x^2 + x + 2 - \frac{7}{x-4}$ (C) $2x^2 + x - 2 + \frac{7}{x-4}$
 (D) $2x^2 - x - 2 + \frac{7}{x-4}$ (E) $2x^2 - x + 2 - \frac{7}{x-4}$

17. If $f(x) = x^3 - x^2$ and $g(x) = x + 1$, what is $f(g^{-1}(\frac{1}{x}))$?
- (A) $-2 + \frac{5}{x} - \frac{4}{x^2} + \frac{1}{x^3}$ (B) $-\frac{1}{x} - \frac{4}{x^2} + \frac{1}{x^3}$
 (C) $-1 + \frac{5}{x} - \frac{4}{x^2} + \frac{1}{x^3}$ (D) $-2 + 5x - 4x^2 + x^3$ (E) $-x - 4x^2 + 1x^3$

18. Consider the graph of $f(x) = \frac{4(x^2-1)}{x^3-1}$ and the following three statements.

- (1) The y -intercept is 4.
 (2) The x -intercept is 1
 (3) There is a vertical asymptote at $x = 1$.

Which of the following is true?

- (A) Only statement (1) is true.
 (B) Only statement (2) is true.
 (C) Only statement (3) is true.
 (D) Exactly two of the statements (1), (2), (3) are true.
 (E) All three statements (1), (2), and (3) are true.

19. Consider the functions $f(x) = \frac{1}{x+2}$, and $g(x) = \frac{1}{x+3}$ and the following three statements.

- (1) Both $f(x)$ and $g(x)$ have horizontal asymptotes at $y = 0$.
 (2) A one-unit shift of $g(x)$ to its right will transform it to $f(x)$
 (3) A one-unit shift of $f(x)$ to its right will transform it to $g(x)$.

Which of the following is true?

- (A) Only statement (1) is true.
 (B) Only statement (2) is true.
 (C) Only statement (3) is true.
 (D) Both of the statements (1) and (3) are correct.
 (E) Both of the statements (1) and (2) are correct.

20. It takes Amanda 2 hours and 3 minutes to paint the walls in her room. She asked her roommate to help her. It would only take them 41 minutes working together to paint their room. If Amanda gets ill on painting day, how long will it take her roommate to paint the entire room by herself?

- (A) 4 hours and 6 minutes (B) 3 hours and 4½ minutes (C) 6 hours and 9 minutes
 (D) 1 hour and 1½ minutes (E) 2 hours and 3 minutes

21. The diameter of a cylinder is one-half of the square root of its height. If the total volume of the cylinder is 4π , what is the height of the cylinder?

- (A) $1/8$ (B) $1/4$ (C) $1/2$ (D) 4 (E) 8

22. What are the vertical asymptotes for the graph $y = \frac{2x + 14}{x^3 + 7x^2 - 4x - 28}$
- (A) $x = -2, x = 2, x = -7$ (B) $x = -2, x = 2$ (C) $x = -2, x = -7$
 (D) $x = 2, x = 7$ (E) $x = -2, x = 2, x = 7$
23. Use the properties of logarithms to combine the following expression into one term: $\log_3\left(\frac{x^2+7x+6}{x^2-2x-24}\right) - \log_3\left(\frac{x^2+19x+78}{x^2+7x+12}\right)$
- (A) $\log_3\left(\frac{x^2+4x+3}{x^2+7x+78}\right)$ (B) $\log_3\left(\frac{x^4+26x^3+217^2+468}{x^4+3x^3-28x-96-144}\right)$ (C) $\log_3\left(\frac{x+1}{x-6}\right)$
 (D) $\log_3\left(\frac{x^2+4x+3}{x^2+7x-78}\right)$ (E) None of the answers (A) through (D) is correct.
24. Solve: $\frac{x^2(3+x)(x+4)}{(x+5)(x-1)} \geq 0$
- (A) $(-\infty, -5) \cup [-4, -3] \cup \{0\} \cup (1, \infty)$ (B) $(-\infty, -5] \cup [-4, -3] \cup \{0\} \cup [1, \infty)$
 (C) $(-\infty, -5) \cup [-4, -3] \cup (1, \infty)$ (D) $(-\infty, -5] \cup [-4, -3] \cup [1, \infty)$
 (E) $(-\infty, -5) \cup (-4, -3) \cup \{0\} \cup (1, \infty)$
25. One group of customers bought 8 deluxe hamburgers, 6 orders of large fries and 6 large colas for \$26.10. A second group ordered 10 deluxe hamburgers, 6 large fries, and 8 large colas for \$31.60. A third group of customers ordered 3 deluxe hamburgers, 2 large fries, and 4 large colas for \$10.95. What is the price of 2 deluxe hamburgers, and 1 large cola?
- (A) \$5.50 (B) \$4.85 (C) \$4.70 (D) \$3.55 (E) \$3.85
26. How far apart are the vertices of the quadratic equations $3x^2 - 18x + 20$ and $2x^2 + 24x + 77$?
- (A) 57 (B) 15 (C) 225 (D) 6 (E) 25
27. You have to make a square-bottomed, unlidged box with a height of three inches and a volume of 48 cubic inches. You will be taking a piece of cardboard, cutting three-inch squares from each corner, scoring between the corners, and folding up the edges. What should be the dimensions of the cardboard?
- (A) 16 in x 16 in (B) 10 in x 10 in (C) 2 in x 2 in
 (D) 6 in x 6 in (E) None of the answers (A) through (D) is correct.
28. The cross-section of a river bed is a parabola 40 meters wide with a maximum depth of 4 meters. Ships traveling down the river need to use river markers so as not to run ashore when passing one another. At what distance from the bank would you put the markers in order to mark where the depth is 3 meters?
- (A) 10 m (B) $10\sqrt{3}$ m (C) 1 m
 (D) 3 m (E) None of the answers (A) through (D) is correct.

29. A car usually depreciates at a yearly rate of $r = 1 - \left(\frac{x}{\text{sold price}}\right)^{\frac{1}{n}}$ with n representing the age of the car and x the present value of the n -year old car. Which of the following expressions represents how much a 9 year old Cadillac is worth if it sold new for \$32,000 and depreciates at a rate of 6.5% per year?

- (A) $x = 32,000(1 - 6.5)^9$ (B) $x = 1 - \left(\frac{6.5}{32000}\right)^{\frac{1}{9}}$ (C) $x = 1 - \left(\frac{9}{32000}\right)^{\frac{1}{6.5}}$
(D) $x = 32000(1 - 0.065)^{\frac{1}{9}}$ (E) $x = 32000(1 - 0.065)^9$

30. A first-time home buyer purchases a house for \$100,000. If the value of the house decreases by $ki\%$ during year k , find the value of i if the value of the house is \$37,500 after two years.

- (A) .05 (B) .25 (C) .5 (D) 5 (E) 25