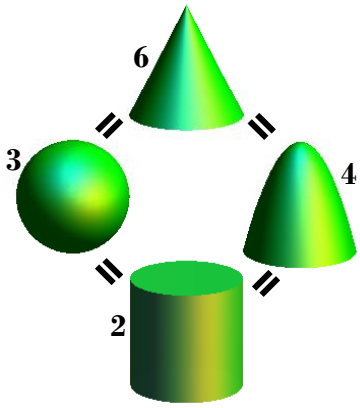
2013
High School Math Contest

Algebra II Exam



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Do's and Don'ts:

- **Do NOT** open this booklet until you are instructed to do so.
- **NO Calculators** (**Or other electronic devices**)
- Contestants with electronic devices (on or off) will be **disqualified!**
  And their **team** will be **disqualified!**
  (The other team members will continue to participate in the individual contest.)

DIRECTIONS:

- **The Score Card:**
  - For the ALGEBRA II exam, you must use the scorecard highlighted in **Yellow**.
  - Write:
    - Your **NAME** on the "**name line**" (of course).
    - "**ALGEBRA II**" on the "**subject line**".
    - Your **SCHOOL** on the "**Date Line**"
- **Clearly mark** ONE bubble using **#2 PENCIL**.
  - Light marks will be counted as unmarked!
  - **Completely erase** any changes.
- You **can** write on this test booklet. (But the test booklet will not be graded.)
- **Tie Breakers**: In case of ties, the person with the least number of *wrong answers* wins.
  (A ***blank*** is better than ***incorrect!***)
- **The Exam:** **30** problems, **70** minutes.
- **WAIT** for the signal to begin.

1. Solve for x: $4^{2x+3} = 8^{3x-5}$

   (A) $\frac{13}{4}$ (B) $\frac{6}{21}$ (C) $\frac{-9}{5}$ (D) $\frac{21}{5}$ (E) $\frac{9}{13}$

2. $(5+3i)(2-i)-(3-4i)(2+i)$ is:

   (A) $5+6i$ (B) $3-4i$ (C) $5-4i$ (D) $3-6i$ (E) $3+6i$

3. Locate all vertical asymptotes of $f(x) = \frac{x^2+8x+15}{x^2-9}$

   (A) $x=3$ (B) $x=-3, x=3$ (C) $y=3$ (D) $y=-3, y=3$
   (E) None of the answers (A) through (D) is correct.

4. The table below shows the number of leaves (L) of four carrots and the mass (M), in grams, of each of the four carrots. According to a line of best fit for the data given in (1), what is the predicted mass of a carrot that has 10 leaves?

$$\hat{M} = -17.5 + 7.6 \cdot L \tag{1}$$

Leaves	7	8	9	10
Mass	35	29	36	58

   (A) -0.5 (B) 0.5 (C) 58 (D) 58.5 (E) 93.5

5. The weight of a body above the surface of the Earth varies inversely as the square of its distance from the center of the Earth. If an astronaut weighs 200 pounds on the surface of the Earth, what will the astronaut weigh 6,000 miles above the Earth's surface, if the distance between the center and the surface of the Earth is 4,000 miles?

   (A) 3.2 pounds (B) 8 pounds (C) 32 pounds (D) 80 pounds (E) 200 pounds

6. Simplify $\frac{2x^2y-5xy^2}{4x^2-25y^2} \div \frac{4x^2+20xy}{2x^2+15xy+25y^2}$

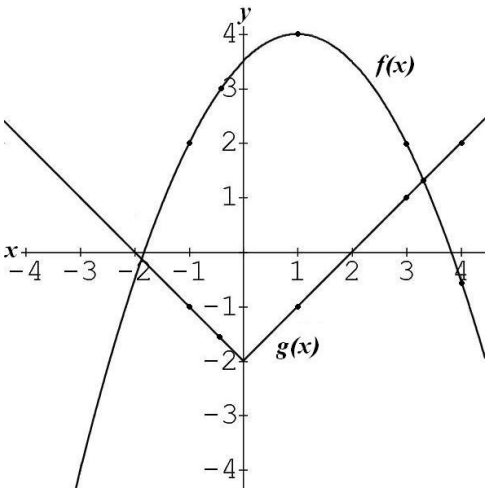
   (A) $\frac{x-2y}{2x+5y}$ (B) $\frac{2x-5y}{4x}$ (C) $\frac{y(2x+5y)}{4(2x-5y)}$ (D) $\frac{4x^2y}{(2x+5y)^2}$ (E) $\frac{y}{4}$

7. If x, y, and z are solutions to the following system of equations, what is $xy+3z$:

$$\begin{cases} x-2y+3z &= 7 \\ 2x+y+z &= 4 \\ -3x+2y-2z &= -10 \end{cases}$$

   (A) 1 (B) -5 (C) -1 (D) 0 (E) 5

8. Refer to the graph below:



What is $f(g(1))$?

(A) 1.25 (B) 3 (C) -1 (D) 2 (E) -1.5

9. Simplify $\dfrac{1-x}{1-\frac{1}{1+\frac{1}{x}}}$

(A) $x^2 - x$ (B) $x - x^2$ (C) $1 + x^2$ (D) $x^2 - 1$ (E) $1 - x^2$

10. Suppose $f(x) = \dfrac{x-2}{x+3}$ and $g(x) = x - 4$. If $H(x) = f(g(x))$ what is the domain of H^{-1}?

(A) $(-\infty, \infty)$ (B) $(-\infty, -1) \cup (-1, \infty)$ (C) $(-\infty, 1) \cup (1, \infty)$

(D) $(-\infty, -6) \cup (-6, \infty)$ (E) $(-\infty, 6) \cup (6, \infty)$

11. Start with $y = -2(x+3)^2$. If we shift the graph to the right 5 units, down 6 units, and reflect it over the x-axis, for what values of x is the resulting function greater than zero?

(A) $(-\infty, 1] \cup [3, \infty)$ (B) $(1, 3)$ (C) $(-\infty, \infty)$ (D) $(-\infty, 1) \cup (3, \infty)$ (E) $[1, 3]$

12. The square of the difference of two numbers m and n is 25, and their product is 12. What is $m^2 + n^2$?

(A) 49 (B) 38 (C) 25 (D) 16 (E) None of the answers (A) through (D) is correct.

13. Given matrices A and B, find AB, where

$$A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} \text{ and } B = \begin{bmatrix} 2 & 3 \\ 1 & 4 \end{bmatrix}$$

(A) $\begin{bmatrix} 4 & 11 \\ 10 & 25 \end{bmatrix}$ (B) $\begin{bmatrix} 4 & 10 \\ 11 & 25 \end{bmatrix}$ (C) $\begin{bmatrix} 2 & 6 \\ 3 & 16 \end{bmatrix}$ (D) $\begin{bmatrix} 2 & 3 \\ 6 & 16 \end{bmatrix}$ (E) $\begin{bmatrix} 6 & 11 \\ 10 & 25 \end{bmatrix}$

14. Suppose $f(x) = x^3 + 6x^2 - 4x - 24$. Find the product of the largest and smallest roots.

(A) 24 (B) -12 (C) -24 (D) -8 (E) 12

15. Suppose a rectangular plot 40 m by 20 m is to contain a rectangular pool with a walkway 2 m wide around the pool. The ratio of the pool area to walkway area is, approximately:

(A) 2.40 (B) 2.57 (C) 3.33 (D) 3.57 (E) 4.80

16. Write $\dfrac{3+i}{4-7i}$ in the form $a + bi$?

(A) $a = \dfrac{1}{13}, b = \dfrac{5}{13}$ (B) $a = \dfrac{-19}{33}, b = \dfrac{-25}{33}$ (C) $a = \dfrac{19}{65}, b = \dfrac{5}{13}$

(D) $a = \dfrac{-5}{33}, b = \dfrac{-25}{33}$ (E) $a = \dfrac{5}{33}, b = \dfrac{-25}{33}$

17. Suppose you win a lottery and suddenly have $\$100,000$ to invest. You decide to put part of the money in a CD (certificate of deposit) which pays 2% annual interest, and put the rest in a savings account which pays 1% annual interest. How much should you put in the CD to earn exactly \$1400 per year, assuming you remove and use the \$1400 each year?

(A) $\$10,000$ (B) $\$20,000$ (C) $\$30,000$ (D) $\$40,000$ (E) $\$60,000$

18. Consider the following statements regarding the graph of $f(x) = \dfrac{4(x^3-1)}{x^2-1}$.

P: The y-intercept is 4.

Q: There is a vertical asymptote at $x = -1$.

R: There is a vertical asymptote at $x = 1$.

Which of the following is true?

(A) Only statement P is true.
(B) Only statement Q is true.
(C) Only statement R is true.
(D) Exactly two of the statements P, Q, and R are true.
(E) All three statements P, Q, and R are true.

19. Solve for x: $\log_4(2x+16) - \log_4(x+7) = 1$

(A) $x = -5$ (B) $x = -6$ (C) $x = -19$ (D) $x = 13$ (E) $x = -9$

20. The diameter of a cylinder is one-half of the square of its height. If the total volume of the cylinder is $\frac{k}{30}\pi$, where k represents the number of degrees of an interior angle of an equilateral triangle, what is the height of the cylinder?

(A) $2^{1/5}$ (B) $2^{1/3}$ (C) $32^{1/3}$ (D) 2 (E) 4

21. Find the distance between the vertex of $y = 2x^2-8x+11$ and the center of the circle $(x+5)^2+(y-4)^2 = 25$

(A) 5 (B) $2\sqrt{17}$ (C) $5\sqrt{2}$ (D) $\sqrt{58}$ (E) 25

22. If $x+1$ is a factor of $x^4 + 2x^3 + kx^2 + 3x + 4k$, what is k?

(A) $\frac{2}{3}$ (B) $\frac{4}{3}$ (C) $\frac{4}{5}$ (D) $\frac{-4}{3}$ (E) $\frac{-4}{5}$

23. Find all real x satisfying $4\Big||x| - 5\Big| - 3 \geq 10$

(A) $\left(-\infty, \frac{5}{4}\right] \cup \left[\frac{-3}{4}, \frac{3}{4}\right] \cup \left[\frac{5}{4}, \infty\right)$

(B) $\left(-\infty, \frac{-25}{4}\right] \cup \left[\frac{-3}{4}, \frac{3}{4}\right] \cup \left[\frac{25}{4}, \infty\right)$

(C) $\left(-\infty, \frac{-17}{4}\right] \cup \left[\frac{-5}{4}, \frac{5}{4}\right] \cup \left[\frac{17}{4}, \infty\right)$

(D) $\left(-\infty, \frac{-21}{4}\right] \cup \left[\frac{-9}{4}, \frac{9}{4}\right] \cup \left[\frac{21}{4}, \infty\right)$

(E) $\left(-\infty, \frac{-33}{4}\right] \cup \left[\frac{-7}{4}, \frac{7}{4}\right] \cup \left[\frac{33}{4}, \infty\right)$

24. Town A and town B are 200 miles apart. Car A leaves town A at noon, and drives toward town B at 40 mph. At 1pm, car B leaves town B at 60 mph, driving along the same road toward town A. If car B stops for lunch from 1:30-2:00pm, at what time do the cars meet?

(A) 3:30 pm (B) 2:42 pm (C) 2:24 pm (D) 2:54 pm (E) 3:00 pm

25. If $f(x) = \sqrt{x}$ and $h \neq 0$, what is $\frac{f(x+h) - f(x)}{h}$?

(A) $\frac{1}{\sqrt{2x-h}}$ (B) $\frac{1}{\sqrt{x+h}+\sqrt{x}}$ (C) $\frac{1}{\sqrt{x+h}-\sqrt{x}}$ (D) $\frac{1}{2\sqrt{x}}$ (E) $\frac{1}{\sqrt{2x+h}}$

26. Find the equation of the line perpendicular to $2x + 4y = 5$ that passes through the center of the circle $x^2 + 4x + y^2 + 6y - 16 = 0$.

(A) $y = 2x + 1$ (B) $y = \frac{-1}{2}x - 4$ (C) $y = 2x - 1$ (D) $y = \frac{-1}{2}x + 4$ (E) $y = -2x - 1$

27. If $f(x) = \log_x(16)$, and $g(x) = 4f^{-1}(x)$, what is $(f \circ g)(2))$?

(A) 1 (B) 2 (C) 4 (D) 8 (E) 16

28. If you place \$0.01 on the first square of a chessboard, \$0.02 on the second square, \$0.04 on the third, and so on, continuing to double the amount until all 64 squares are covered, how much money will be on the board?

(A) $\frac{2^{62}}{25}$ dollars (B) $\frac{2^{63}}{25}$ dollars (C) $\frac{2^{64}}{25}$ dollars (D) $\left(\frac{2^{62}}{25} - 1\right)$ dollars and 99 cents
(E) $\left(\frac{2^{63}}{25} - 1\right)$ dollars and 99 cents

29. For what values of k will the function $f(x) = \dfrac{x^2 + 4x + k}{x^3 - 3x^2 - 9x + 27}$ have at most one zero?

(A) $k \geq 4$ or $k = -3$ or $k = 3$
(B) $k \geq 4$ or $k = -21$ or $k = 3$
(C) $k \geq 4$ or $k = -21$ or $k = -3$
(D) $k \geq 4$ or $k = -21$ or $k = -3$ or $k = 3$
(E) None of the answers (A) through (D) is correct.

30. Expand the following logarithm: $\log_5\left(\sqrt[3]{\dfrac{(x-5)^4x^2}{25\sqrt{x}}}\right)$

(A) $\frac{11}{6}\log_5(x) - 2$ (B) $\frac{13}{6}\log_5(x) - 2$ (C) $\frac{4}{3}\log_5(x-5) + \frac{5}{6}\log_5(x) - \frac{2}{3}$
(D) $\frac{4}{3}\log_5(x-5) + \frac{1}{2}\log_5(x) - \frac{2}{3}$ (E) None of the answers (A) through (D) is correct.